

## Study Guide

## Key Concepts

## Geometric Mean (Lesson 8-1)

- For two positive numbers  $a$  and  $b$ , the geometric mean is the positive number  $x$  where  $a : x = x : b$  is true.

## Pythagorean Theorem (Lesson 8-2)

- Let  $\triangle ABC$  be a right triangle with right angle  $C$ . Then  $a^2 + b^2 = c^2$ .

## Special Right Triangles (Lesson 8-3)

- The measures of the sides of a  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle are  $x$ ,  $x$ , and  $x\sqrt{2}$ .
- The measures of the sides of a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle are  $x$ ,  $2x$ , and  $x\sqrt{3}$ .

## Trigonometry (Lesson 8-4)

- $\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

## Angles of Elevation and Depression (Lesson 8-5)

- An angle of elevation is the angle formed by a horizontal line and the line of sight to an object above.
- An angle of depression is the angle formed by a horizontal line and the line of sight to an object below.

## Laws of Sines and Cosines (Lesson 8-6)

Let  $\triangle ABC$  be any triangle.

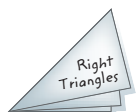
- Law of Sines:  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Law of Cosines:  $a^2 = b^2 + c^2 - 2bc \cos A$   
 $b^2 = a^2 + c^2 - 2ac \cos B$   
 $c^2 = a^2 + b^2 - 2ab \cos C$

## Vectors (Lesson 8-7)

- A vector is a quantity with both magnitude and direction.

## FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



## Key Vocabulary



- angle of depression (p. 580)
- angle of elevation (p. 580)
- component form (p. 602)
- cosine (p. 568)
- direction (p. 600)
- geometric mean (p. 537)
- inverse cosine (p. 571)
- inverse sine (p. 571)
- inverse tangent (p. 571)
- Law of Cosines (p. 589)
- Law of Sines (p. 588)
- magnitude (p. 600)
- Pythagorean triple (p. 548)
- resultant (p. 601)
- sine (p. 568)
- standard position (p. 602)
- tangent (p. 568)
- trigonometric ratio (p. 568)
- trigonometry (p. 568)
- vector (p. 600)

## Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

- The arithmetic mean of two numbers is the positive square root of the product of the numbers.
- Extended ratios can be used to compare three or more quantities.
- To find the length of the hypotenuse of a right triangle, take the square root of the difference of the squares of the legs.
- An angle of elevation is the angle formed by a horizontal line and an observer's line of sight to an object below the horizon.
- The sum of two vectors is the resultant.
- Magnitude is the angle a vector makes with the x-axis.
- A vector is in standard position when the initial point is at the origin.
- The component form of a vector describes the vector in terms of change in  $x$  and change in  $y$ .
- The Law of Sines can be used to find an angle measure when given three side lengths.
- A trigonometric ratio is a ratio of the lengths of two sides of a right triangle.

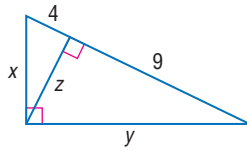


# Lesson-by-Lesson Review

## 8-1 Geometric Mean

Find the geometric mean between each pair of numbers.

11. 9 and 4
12.  $\sqrt{20}$  and  $\sqrt{80}$
13.  $\frac{8\sqrt{2}}{3}$  and  $\frac{4\sqrt{2}}{3}$
14. Find  $x$ ,  $y$ , and  $z$ .



15. **DANCES** Mike is hanging a string of lights on his barn for a square dance. Using a book to sight the top and bottom of the barn, he can see he is 15 feet from the barn. If his eye level is 5 feet from the ground, how tall is the barn?

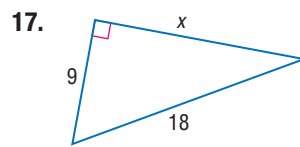
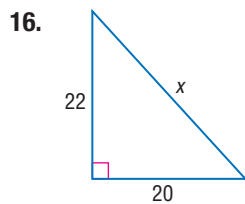
### Example 1

Find the geometric mean between 10 and 15.

$$\begin{aligned}
 x &= \sqrt{ab} && \text{Definition of geometric mean} \\
 &= \sqrt{10 \cdot 15} && a = 10 \text{ and } b = 15 \\
 &= \sqrt{(5 \cdot 2) \cdot (3 \cdot 5)} && \text{Factor.} \\
 &= \sqrt{25 \cdot 6} && \text{Associative Property} \\
 &= 5\sqrt{6} && \text{Simplify.}
 \end{aligned}$$

## 8-2 The Pythagorean Theorem and Its Converse

Find  $x$ .

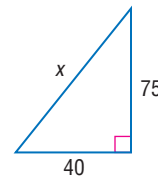


Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer.

18. 7, 24, 25
19. 13, 15, 16
20. 65, 72, 88
21. **SWIMMING** Alexi walks 27 meters south and 38 meters east to get around a lake. Her sister swims directly across the lake. How many meters to the nearest tenth did Alexi's sister save by swimming?

### Example 2

Find  $x$ .



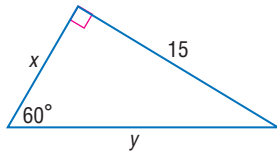
The side opposite the right angle is the hypotenuse, so  $c = x$ .

$$\begin{aligned}
 a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\
 40^2 + 75^2 &= x^2 && a = 40 \text{ and } b = 75 \\
 7225 &= x^2 && \text{Simplify.} \\
 \sqrt{7225} &= x && \text{Take the positive square root of each side.} \\
 85 &= x && \text{Simplify.}
 \end{aligned}$$

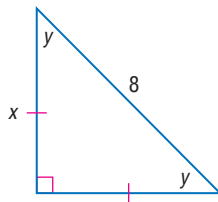
8-3 Special Right Triangles

Find  $x$  and  $y$ .

22.



23.



24. **CLIMBING** Jason is adding a climbing wall to his little brother's swing-set. If he starts building 5 feet out from the existing structure, and wants it to have a  $60^\circ$  angle, how long should the wall be?

Example 3

Find  $x$  and  $y$ .

The measure of the third angle in this triangle is  $90 - 60$  or  $30$ . This is a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle.

$h = 2s$   $30^\circ$ - $60^\circ$ - $90^\circ$  Triangle Theorem

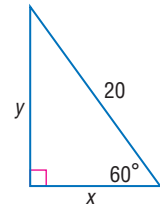
$20 = 2x$  **Substitute.**

$10 = x$  **Divide.**

Now find  $y$ , the length of the longer leg.

$l = s\sqrt{3}$   $30^\circ$ - $60^\circ$ - $90^\circ$  Triangle Theorem

$y = 10\sqrt{3}$  **Substitute.**



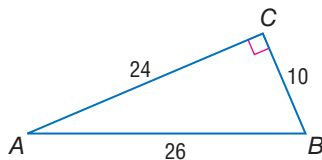
8-4 Trigonometry

Express each ratio as a fraction and as a decimal to the nearest hundredth.

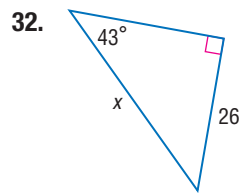
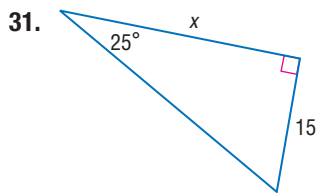
25.  $\sin A$     26.  $\tan B$

27.  $\sin B$     28.  $\cos A$

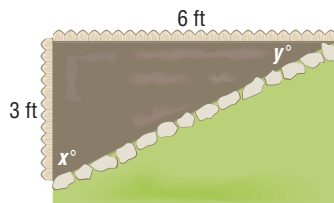
29.  $\tan A$     30.  $\cos B$



Find  $x$ .



33. **GARDENING** Sofia wants to put a flower bed in the corner of her yard by laying a stone border that starts 3 feet from the corner of one fence and ends 6 feet from the corner of the other fence. Find the angles,  $x$  and  $y$ , the fence make with the border.



Example 4

Express each ratio as a fraction and as a decimal to the nearest hundredth.

a.  $\sin L$

$\sin L = \frac{5}{13}$  or about 0.38

$\sin L = \frac{\text{opp}}{\text{hyp}}$

b.  $\cos L$

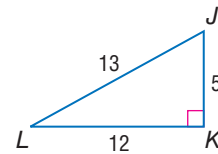
$\cos L = \frac{12}{13}$  or about 0.92

$\cos L = \frac{\text{adj}}{\text{hyp}}$

c.  $\tan L$

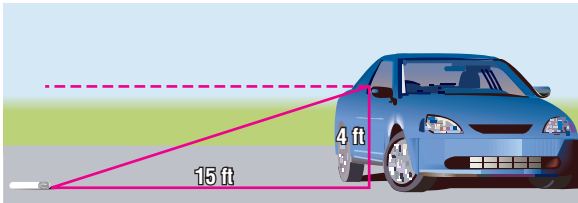
$\tan L = \frac{5}{12}$  or 0.42

$\tan L = \frac{\text{opp}}{\text{adj}}$



## 8-5 Angles of Elevation and Depression

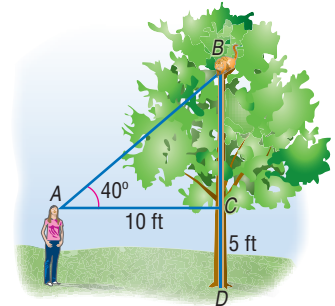
34. **JOBS** Tom delivers papers on a rural route from his car. If he throws a paper from a height of 4 feet, and it lands 15 feet from the car, at what angle of depression did he throw the paper to the nearest degree?



35. **TOWER** There is a cell phone tower in the field across from Jen's house. If Jen walks 50 feet from the tower, and finds the angle of elevation from her position to the top of the tower to be  $60^\circ$ , how tall is the tower?

### Example 5

Sarah's cat climbed up a tree. If she sights her cat at an angle of elevation of  $40^\circ$ , and her eyes are 5 feet off the ground, how high up from the ground is her cat?



To find the how high the cat is up the tree, find  $CB$ .

$$\tan 40 = \frac{CB}{10}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$

$$10(\tan 40) = CB$$

$$8.4 = CB$$

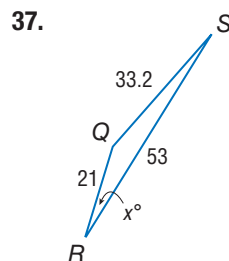
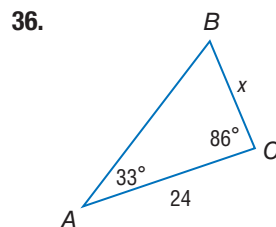
Multiply each side by 10.

Simplify.

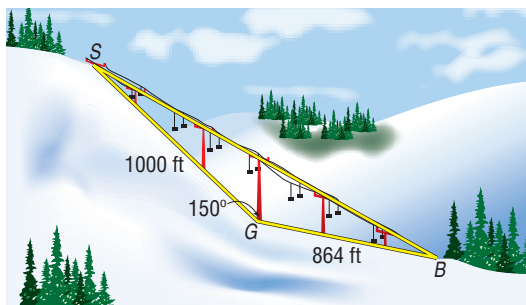
Since Sarah's eyes are 5 feet from the ground, add 5 to 8.4. Sarah's cat is 13.4 feet up.

## 8-6 The Law of Sines and Law of Cosines

Find  $x$ . Round angle measures to the nearest degree and side measures to the nearest tenth.

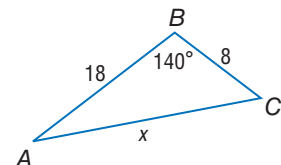


38. **SKIING** At Crazy Ed's Ski resort, Ed wants to put in another ski lift for the skiers to ride from the base to the summit of the mountain. The run over which the ski lift will go is represented by the figure below. The length of the lift is represented by  $\overline{SB}$ . If Ed needs twice as much cable as the length of  $\overline{SB}$ , how much cable does he need?



### Example 6

Find  $x$ . Round to the nearest tenth.



We are given the measures of two sides and their included angle, so use the Law of Cosines.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Law of Cosines

$$x^2 = 8^2 + 18^2 - 2(8)(18) \cos 140^\circ$$

Substitution

$$x^2 = 388 - 288 \cos 140^\circ$$

Simplify.

$$x = \sqrt{388 - 288 \cos 140^\circ} \approx 24.7$$

Take the square root of each side.

### Example 7

Find  $x$ . Round to the nearest tenth.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

Law of Sines

$$\frac{\sin 60}{12} = \frac{\sin x}{11}$$

Substitution

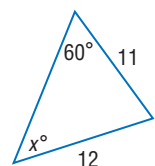
$$11 \sin 60^\circ = 12 \sin x$$

Cross Products Property

$$\frac{11 \sin 60}{12} = \sin x$$

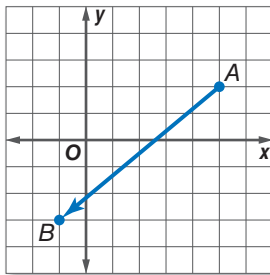
Divide each side by 12.

$$x = \sin^{-1} \frac{11 \sin 60}{12} \text{ or about } 52.5^\circ$$



8-7 Vectors

39. Write the component form of the vector shown.



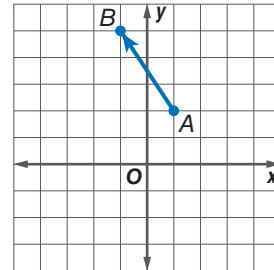
40. Copy the vectors to find  $\vec{a} + \vec{b}$ .



41. Given that  $\vec{s}$  is  $\langle 2, -6 \rangle$  and  $\vec{t}$  is  $\langle -10, 7 \rangle$ , find the component form of  $\vec{s} + \vec{t}$ .

Example 8

Find the magnitude and direction of  $\vec{AB}$  for  $A(1, 2)$  and  $B(-1, 5)$ .



Use the Distance Formula to find the magnitude.

$$\begin{aligned} \overline{AB} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-1 - 1)^2 + (5 - 2)^2} && \text{Substitute.} \\ &= \sqrt{13} \text{ or about } 3.6 && \text{Simplify.} \end{aligned}$$

Draw a right triangle with hypotenuse  $\vec{AB}$  and acute angle  $A$ .

$$\tan A = \left| \frac{5 - 2}{-1 - 1} \right| \text{ or } \frac{3}{2} \quad \tan = \frac{\text{opp}}{\text{adj}}; \text{ length cannot be negative.}$$

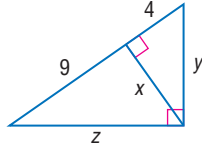
$$m\angle A = \tan^{-1} \left( -\frac{3}{2} \right) \quad \text{Def. of inverse tangent}$$

$$\approx -56.3 \quad \text{Use a calculator.}$$

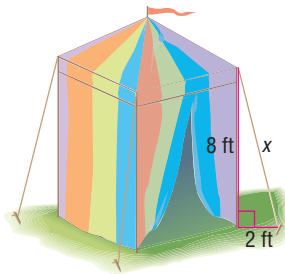
The direction of  $\vec{AB}$  is  $180 - 56.3$  or  $123.7^\circ$ .

Find the geometric mean between each pair of numbers.

- 7 and 11
- 12 and 9
- 14 and 21
- $4\sqrt{3}$  and  $10\sqrt{3}$
- Find  $x$ ,  $y$ , and  $z$ .



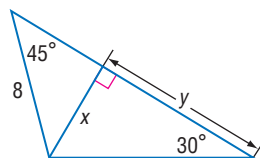
- FAIRS** Blake is setting up his tent at a renaissance fair. If the tent is 8 feet tall, and the tether can be staked no more than two feet from the tent, how long should the tether be?



Use a calculator to find the measure of  $\angle R$  to the nearest tenth.

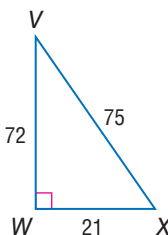
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- Find  $x$  and  $y$ .



Express each ratio as a fraction and as a decimal to the nearest hundredth.

- $\cos X$
- $\tan V$
- $\tan X$
- $\sin V$

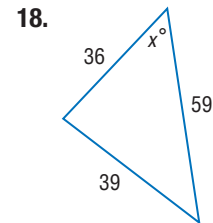
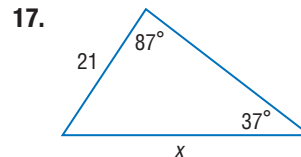


Find the magnitude and direction of each vector.

- $\vec{JK}$ :  $J(-6, -4)$  and  $K(-10, -4)$
- $\vec{RS}$ :  $R(1, 0)$  and  $S(-2, 3)$

- SPACE** Anna is watching a space shuttle launch 6 miles from Cape Canaveral in Florida. When the angle of elevation from her viewing point to the shuttle is  $80^\circ$ , how high is the shuttle, if it is going straight up?

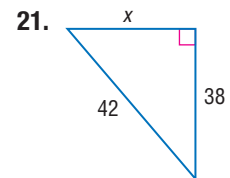
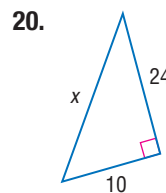
Find  $x$ . Round angle measures to the nearest degree and side measures to the nearest tenth.



- MULTIPLE CHOICE** Which of the following is the length of the leg of a  $45^\circ-45^\circ-90^\circ$  triangle with a hypotenuse of 20?

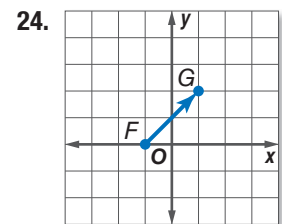
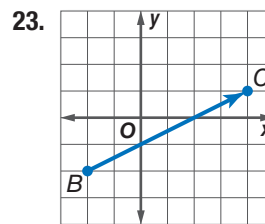
- A 10                      C 20  
B  $10\sqrt{2}$               D  $20\sqrt{2}$

Find  $x$ .



- WHALE WATCHING** Isaac is looking through binoculars on a whale watching trip when he notices a sea otter in the distance. If he is 20 feet above sea level in the boat, and the angle of depression is  $30^\circ$ , how far away from the boat is the otter to the nearest foot?

Write the component form of each vector.



- Solve  $\triangle FGH$ . Round to the nearest degree.

