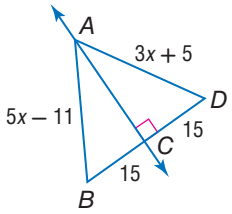
CHAPTER 5

Mid-Chapter Quiz

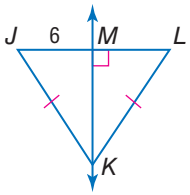
Lessons 5-1 through 5-3

Find each measure. (Lesson 5-1)

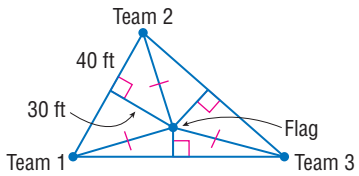
1. *AB*



2. *JL*

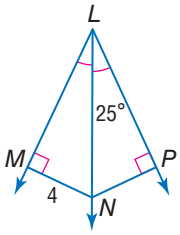


3. CAMP Camp Onawatchi ends with a game of capture the flag. If the starting locations of three teams are shown in the diagram below, with the flag at a point equidistant from each team's base, how far from each base is the flag in feet? (Lesson 5-1)

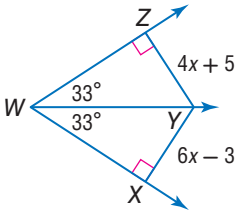


Find each measure. (Lesson 5-1)

4. $\angle MNP$



5. *XY*

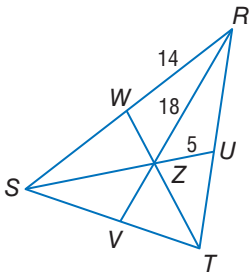


In $\triangle RST$, Z is the centroid and $RZ = 18$. Find each length. (Lesson 5-2)

6. *ZV*

7. *SZ*

8. *SR*



COORDINATE GEOMETRY **Find the coordinates of the centroid of each triangle with the given vertices.** (Lesson 5-2)

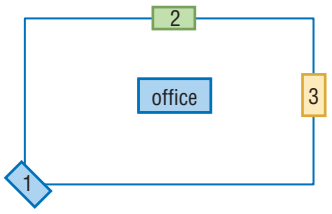
9. $A(1, 7)$, $B(4, 2)$, $C(7, 7)$

10. $X(-11, 0)$, $Y(-11, -8)$, $Z(-1, -4)$

11. $R(-6, 4)$, $S(-2, -2)$, $T(2, 4)$

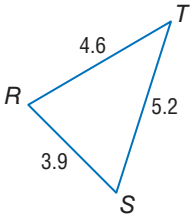
12. $J(-5, 5)$, $K(-5, -1)$, $L(1, 2)$

13. ARCHITECTURE An architect is designing a high school building. Describe how to position the central office so that it is at the intersection of each hallway connected to the three entrances to the school. (Lesson 5-2)

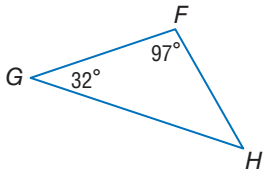


List the angles and sides of each triangle in order from smallest to largest. (Lesson 5-3)

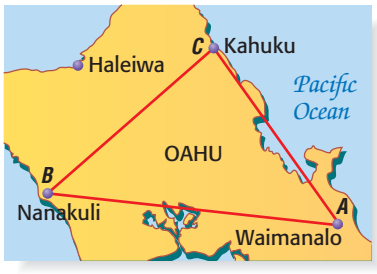
14.



15.



16. VACATION Kailey plans to fly over the route marked on the map of Hawaii below. (Lesson 5-3)

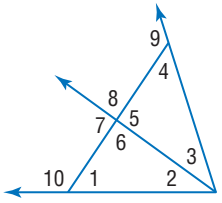


a. If $m\angle A = 2 + m\angle B$ and $m\angle C = 2(m\angle B) - 14$, what are the measures of the three angles?

b. What are the lengths of Kailey's trip in order of least to greatest?

c. The length of the entire trip is about 68 miles. The middle leg is 11 miles greater than one-half the length of the shortest leg. The longest leg is 12 miles greater than three-fourths of the shortest leg. What are the lengths of the legs of the trip?

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition. (Lesson 5-3)



17. measures less than $m\angle 8$

18. measures greater than $m\angle 3$

19. measures less than $m\angle 10$

EXPLORE

5-4 Geometry Lab: Matrix Logic

Matrix logic uses a rectangular array in which you record what you have learned from clues in order to solve a logic or reasoning problem. Once all the rows and columns are filled, you can deduce the answer.

FOOD **Matt, Abby, Javier, Corey, and Keisha go to an Italian restaurant. Each orders their favorite dish: ravioli, pizza, lasagna, manicotti, or spaghetti. Javier loves ravioli, but Matt does not like pasta dishes. Abby does not like lasagna or manicotti. Corey's favorite dish does not end in the letter i. What does each person order?**

Step 1 Create an appropriate matrix.

Use a 5 × 5 matrix that includes each person's name as the header for each row and their possible favorite foods as the header for each column.

		Favorite Dish				
		ravioli	pizza	lasagna	manicotti	spaghetti
Name	Matt	×	✓	×	×	×
	Abby	×	×			
	Javier	✓	×	×	×	×
	Corey	×	×			
	Keisha	×	×			

Step 2 Use each clue and logical reasoning to fill in the matrix.

- Since Javier loves ravioli, place a ✓ in Javier's row under ravioli and an × in every other cell in his row. Since only one person likes each dish, you can place an × in every other cell in the ravioli column.

- Since Matt does not like pasta, you know that Matt cannot like manicotti, ravioli, lasagna, or spaghetti, which are all pasta dishes. Therefore, Matt must like pizza. Place a ✓ in Matt's row under pizza. Place an × in every other cell in Matt's row and in every other cell in the pizza column.

		Favorite Dish				
		ravioli	pizza	lasagna	manicotti	spaghetti
Name	Matt	×	✓	×	×	×
	Abby	×	×	×	×	✓
	Javier	✓	×	×	×	×
	Corey	×	×			×
	Keisha	×	×			×

- Since Abby does not like lasagna or manicotti, place an × in Abby's row under lasagna and manicotti. This leaves only spaghetti without an × for Abby's row. Therefore, you can conclude that Abby must like spaghetti. Place a ✓ in that cell and an × in every other cell in the spaghetti column.

- From the matrix, you can see that Corey's favorite dish must be either lasagna or manicotti. However, since Corey's favorite dish does not end in the letter i, you can conclude that Corey must like lasagna. In Corey's row, place a ✓ under lasagna and an × under manicotti.

		Favorite Dish				
		ravioli	pizza	lasagna	manicotti	spaghetti
Name	Matt	×	✓	×	×	×
	Abby	×	×	×	×	✓
	Javier	✓	×	×	×	×
	Corey	×	×	✓	×	×
	Keisha	×	×	×	✓	×

- This leaves only one empty cell in Keisha's row, so you can conclude that her favorite dish is manicotti.

Step 3 Use your matrix to state the answer to the problem.

From the matrix you can state that Matt orders pizza, Abby orders spaghetti, Javier orders ravioli, Corey orders lasagna, and Keisha orders manicotti.

Matrix Logic *Continued*

Exercises

Use a matrix to solve each problem.

1. **SPORTS** Trey, Nathan, Parker, and Chen attend the same school. Each participates in a different school sport: basketball, football, track, or tennis. Use the following clues to determine in which sport each student participates.

   - Nathan does not like track or basketball.
   - Trey does not participate in football or tennis.
   - Parker prefers an indoor winter sport.
   - Chen scored four touchdowns in the final game of the season.

2. **FAMILY** The Martin family has five children. Use the following clues to determine in what order the children were born.

   - Grace is older than Hannah.
   - Thomas is younger than Sarah.
   - Hannah is older than Thomas and Samuel.
   - Samuel is older than Thomas.
   - Sarah is older than Grace.

3. **PETS** Alejandra, Tamika, and Emily went to a pet store. Each girl chose a different pet to adopt: a dog, a rabbit, or a cat. Each girl named her pet Sweet Pea, Zuzu, or Roscoe. Use the following clues and the matrix shown to determine the animal each girl adopted and what name she gave her pet.

   - The girl who adopted a dog did not name it Sweet Pea.
   - Tamika's pet, who she named Zuzu, is not the type of animal that hops.
   - Roscoe, who is not a cat, was adopted by Emily.
   - The rabbit was not adopted by Alejandra.

		Pet			Pet Name		
		dog	rabbit	cat	Sweet Pea	Zuzu	Roscoe
Names	Alejandra						
	Tamika						
	Emily						
Pet Name	Sweet Pea						
	Zuzu						
	Roscoe						

4. **GEOMETRY** Kasa, Marcus, and Jason each drew a triangle, no two of which share the same side or angle classification. Use the following clues to determine what type of triangle each person has drawn.

   - Kasa did not draw an equilateral triangle.
   - Marcus' triangle has one angle that measures 25 and another that measures 65.
   - Jason drew a triangle with at least one pair of congruent sides.
   - The obtuse triangle has two congruent angles.

LESSON 5-4

Indirect Proof

Then

- You wrote paragraph, two-column, and flow proofs.

Now

1. Write indirect algebraic proofs.
2. Write indirect geometric proofs.

Why?

- **Matthew:** "I'm almost positive Friday is not a teacher work day, but I can't prove it."
- **Kim:** "Let's assume that Friday *is* a teacher work day. What day is our next Geometry test?"
- **Ana:** "Hmmm . . . according to the syllabus, it's this Friday. But we don't have tests on teacher work days—we're not in school."
- **Jamal:** "Exactly—so that proves it! This Friday can't be a teacher work day."

Digital Vision/Punchstock

New Vocabulary

indirect reasoning
indirect proof
proof by contradiction

Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

2 Reason abstractly and quantitatively.

1 Indirect Algebraic Proof

The proofs you have written have been *direct proofs*—you started with a true hypothesis and proved that the conclusion was true. In the example above, the students used **indirect reasoning**, by assuming that a conclusion was false and then showing that this assumption led to a contradiction.

In an **indirect proof** or **proof by contradiction**, you temporarily assume that what you are trying to prove is false. By showing this assumption to be logically impossible, you prove your assumption false and the original conclusion true. Sometimes this is called *proof by negation*.

KeyConcept How to Write an Indirect Proof

Step 1 Identify the conclusion you are asked to prove. Make the assumption that this conclusion is false by assuming that the opposite is true.

Step 2 Use logical reasoning to show that this assumption leads to a contradiction of the hypothesis, or some other fact, such as a definition, postulate, theorem, or corollary.

Step 3 Point out that since the assumption leads to a contradiction, the original conclusion, what you were asked to prove, must be true.

Example 1 State the Assumption for Starting an Indirect Proof

State the assumption necessary to start an indirect proof of each statement.

a. If 6 is a factor of n, then 2 is a factor of n.

The conclusion of the conditional statement is *2 is a factor of n*. The negation of the conclusion is *2 is not a factor of n*.

b. $\angle 3$ is an obtuse angle.

If *$\angle 3$ is an obtuse angle* is false, then *$\angle 3$ is not an obtuse angle* must be true.

GuidedPractice

1A. $x > 5$

1B. $\triangle XYZ$ is an equilateral triangle.

Indirect proofs can be used to prove algebraic concepts.

Example 2 Write an Indirect Algebraic Proof

Write an indirect proof to show that if $-3x + 4 > 16$, then $x < -4$.

Given: $-3x + 4 > 16$

Prove: $x < -4$

Step 1 **Indirect Proof:**
The negation of $x < -4$ is $x \geq -4$. So, assume that $x > -4$ or $x = -4$ is true.

Step 2 Make a table with several possibilities for x assuming $x > -4$ or $x = -4$.

x	-4	-3	-2	-1	0
$-3x + 4$	16	13	10	7	4

When $x > -4$, $-3x + 4 < 16$ and when $x = -4$, $-3x + 4 = 16$.

Step 3 In both cases, the assumption leads to the contradiction of the given information that $-3x + 4 > 16$. Therefore, the assumption that $x \geq -4$ must be false, so the original conclusion that $x < -4$ must be true.

ReadingMath
Contradiction
A contradiction is a principle of logic stating that an assumption cannot be both *A* and the opposite of *A* at the same time.

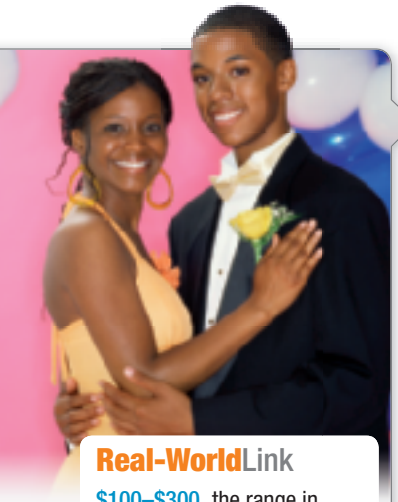
GuidedPractice

Write an indirect proof of each statement.

2A. If $7x > 56$, then $x > 8$.

2B. If $-c$ is positive, then c is negative.

Indirect reasoning and proof can be used in everyday situations.



Real-WorldLink
$100–$300 the range in price of a girl's prom dress
$75–$125 the range in cost for a tuxedo rental
around $150 the cost of a fancy dinner for two
$100–$200 the range in cost of prom tickets per couple
Source: PromSpot

Real-World Example 3 Indirect Algebraic Proof

PROM COSTS **Javier asked his friend Christopher the cost of his meal and his date's meal when he went to dinner for prom. Christopher could not remember the individual costs, but he did remember that the total bill, not including tip, was over $60. Use indirect reasoning to show that at least one of the meals cost more than $30.**

Let the cost of one meal be x and the cost of the other meal be y.

Step 1 **Given:** $x + y > 60$

Prove: $x > 30$ or $y > 30$

Indirect Proof:
Assume that $x \leq 30$ and $y \leq 30$.

Step 2 If $x \leq 30$ and $y \leq 30$, then $x + y \leq 30 + 30$ or $x + y \leq 60$. This is a contradiction because we know that $x + y > 60$.

Step 3 Since the assumption that $x \leq 30$ and $y \leq 30$ leads to a contradiction of a known fact, the assumption must be false. Therefore, the conclusion that $x > 30$ or $y > 30$ must be true. Thus, at least one of the meals had to cost more than $30.

GuidedPractice

3. TRAVEL Cleavon traveled over 360 miles on his trip, making just two stops. Use indirect reasoning to prove that he traveled more than 120 miles on one leg of his trip.

Stockbyte/Getty Images

Indirect proofs are often used to prove concepts in number theory. In such proofs, it is helpful to remember that you can represent an even number with the expression $2k$ and an odd number with the expression $2k + 1$ for any integer k.

Example 4 Indirect Proofs in Number Theory

Write an indirect proof to show that if $x + 2$ is an even integer, then x is an even integer.

Step 1 **Given:** $x + 2$ is an even integer.

Prove: x is an even integer.

Indirect Proof:
Assume that x is an odd integer. This means that $x = 2k + 1$ for some integer k.

Step 2
$$x + 2 = (2k + 1) + 2 \quad \text{Substitution of assumption}$$
$$= (2k + 2) + 1 \quad \text{Commutative Property}$$
$$= 2(k + 1) + 1 \quad \text{Distributive Property}$$

Now determine whether $2(k + 1) + 1$ is an even or odd integer. Since k is an integer, $k + 1$ is also an integer. Let m represent the integer $k + 1$.

$$2(k + 1) + 1 = 2m + 1 \quad \text{Substitution}$$

So, $x + 2$ can be represented by $2m + 1$, where m is an integer. But this representation means that $x + 2$ is an odd integer, which contradicts the given statement that $x + 2$ is an even integer.

Step 3 Since the assumption that x is an odd integer leads to a contradiction of the given statement, the original conclusion that x is an even integer must be true.

WatchOut!
CCSS Arguments Proof by contradiction and using a counterexample are not the same. A counterexample helps you disprove a conjecture. It cannot be used to prove a conjecture.

GuidedPractice

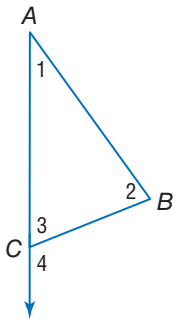
4. Write an indirect proof to show that if the square of an integer is odd, then the integer is odd.

2 Indirect Proof with Geometry

Indirect reasoning can be used to prove statements in geometry, such as the Exterior Angle Inequality Theorem.

Example 5 Geometry Proof

If an angle is an exterior angle of a triangle, prove that its measure is greater than the measure of either of its corresponding remote interior angles.



Step 1 Draw a diagram of this situation. Then identify what you are given and what you are asked to prove.

Given: $\angle 4$ is an exterior angle of $\triangle ABC$.

Prove: $m\angle 4 > m\angle 1$ and $m\angle 4 > m\angle 2$.

Indirect Proof:
Assume that $m\angle 4 \ngtr m\angle 1$ or $m\angle 4 \ngtr m\angle 2$.
In other words, $m\angle 4 \leq m\angle 1$ or $m\angle 4 \leq m\angle 2$.

(continued on the next page)

Step 2 You need only show that the assumption $m\angle 4 \leq m\angle 1$ leads to a contradiction. The argument for $m\angle 4 \leq m\angle 2$ follows the same reasoning.

$m\angle 4 \leq m\angle 1$ means that either $m\angle 4 = m\angle 1$ or $m\angle 4 < m\angle 1$.

Case 1 $m\angle 4 = m\angle 1$

$m\angle 4 = m\angle 1 + m\angle 2$ Exterior Angle Theorem

$m\angle 4 = m\angle 4 + m\angle 2$ Substitution

$0 = m\angle 2$ Subtract $m\angle 4$ from each side.

This contradicts the fact that the measure of an angle is greater than 0, so $m\angle 4 \neq m\angle 1$.

Case 2 $m\angle 4 < m\angle 1$

By the Exterior Angle Theorem, $m\angle 4 = m\angle 1 + m\angle 2$. Since angle measures are positive, the definition of inequality implies that $m\angle 4 > m\angle 1$. This contradicts the assumption that $m\angle 4 < m\angle 1$.

Step 3 In both cases, the assumption leads to the contradiction of a theorem or definition. Therefore, the original conclusion that $m\angle 4 > m\angle 1$ and $m\angle 4 > m\angle 2$ must be true.

StudyTip

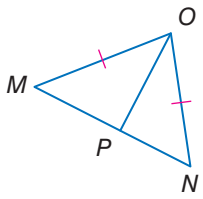
Recognizing Contradictions
Remember that the contradiction in an indirect proof is not always of the given information or the assumption. It can be of a known fact or definition, such as in Case 1 of Example 5—the measure of an angle must be greater than 0.

GuidedPractice

5. Write an indirect proof.

Given: $\overline{MO} \cong \overline{ON}$, $\overline{MP} \ncong \overline{NP}$

Prove: $\angle MOP \ncong \angle NOP$



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **State the assumption you would make to start an indirect proof of each statement.**

1. $\overline{AB} \cong \overline{CD}$

2. $\triangle XYZ$ is a scalene triangle.

3 If $4x < 24$, then $x < 6$.

4. $\angle A$ is not a right angle.

Example 2 **Write an indirect proof of each statement.**

5. If $2x + 3 < 7$, then $x < 2$.

6. If $3x - 4 > 8$, then $x > 4$.

Example 3

7. LACROSSE Christina scored 13 points for her high school lacrosse team during the last six games. Prove that her average points per game was less than 3.

Example 4

8. Write an indirect proof to show that if $5x - 2$ is an odd integer, then x is an odd integer.

Example 5 **Write an indirect proof of each statement.**

9. The hypotenuse of a right triangle is the longest side.

10. If two angles are supplementary, then they both cannot be obtuse angles.

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **State the assumption you would make to start an indirect proof of each statement.**

11. If $2x > 16$, then $x > 8$.

12. $\angle 1$ and $\angle 2$ are not supplementary angles.

13. If two lines have the same slope, the lines are parallel.

14. If the consecutive interior angles formed by two lines and a transversal are supplementary, the lines are parallel.

15. If a triangle is not equilateral, the triangle is not equiangular.

16. An odd number is not divisible by 2.

Example 2 **Write an indirect proof of each statement.**

17 If $2x - 7 > -11$, then $x > -2$.

18. If $5x + 12 < -33$, then $x < -9$.

19. If $-3x + 4 < 7$, then $x > -1$.

20. If $-2x - 6 > 12$, then $x < -9$.

Example 3

21. COMPUTER GAMES Kwan-Yong bought two computer games for just over \$80 before tax. A few weeks later, his friend asked how much each game cost. Kwan-Yong could not remember the individual prices. Use indirect reasoning to show that at least one of the games cost more than \$40.

22. FUNDRAISING Jamila's school is having a Fall Carnival to raise money for a local charity. The cost of an adult ticket to the carnival is \$6 and the cost of a child's ticket is \$2.50. If 375 total tickets were sold and the profit was more than \$1460, prove that at least 150 adult tickets were sold.

Examples 4–5 CCSS **ARGUMENTS** **Write an indirect proof of each statement.**

23. Given: xy is an odd integer.

Prove: x and y are both odd integers.

24. Given: n^2 is even.

Prove: n^2 is divisible by 4.

25. Given: x is an odd number.

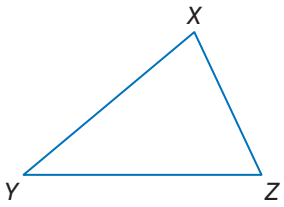
Prove: x is not divisible by 4.

26. Given: xy is an even integer.

Prove: x or y is an even integer.

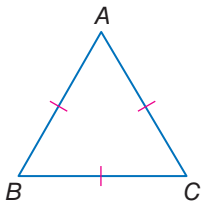
27. Given: $XZ > YZ$

Prove: $\angle X \ncong \angle Y$



28. Given: $\triangle ABC$ is equilateral.

Prove: $\triangle ABC$ is equiangular.



29. In an isosceles triangle neither of the base angles can be a right angle.

30. A triangle can have only one right angle.

31. Write an indirect proof for Theorem 5.10.

32. Write an indirect proof to show that if $\frac{1}{b} < 0$, then b is negative.

33. **BASKETBALL** In basketball, there are three possible ways to score three points in a single possession. A player can make a basket from behind the three-point line, a player may be fouled while scoring a two-point shot and be allowed to shoot one free throw, or a player may be fouled behind the three-point line and be allowed to shoot three free throws. When Katsu left to get in the concession line, the score was 28 home team to 26 visiting team. When she returned, the score was 28 home team to 29 visiting team. Katsu concluded that a player on the visiting team had made a three-point basket. Prove or disprove her assumption using an indirect proof.

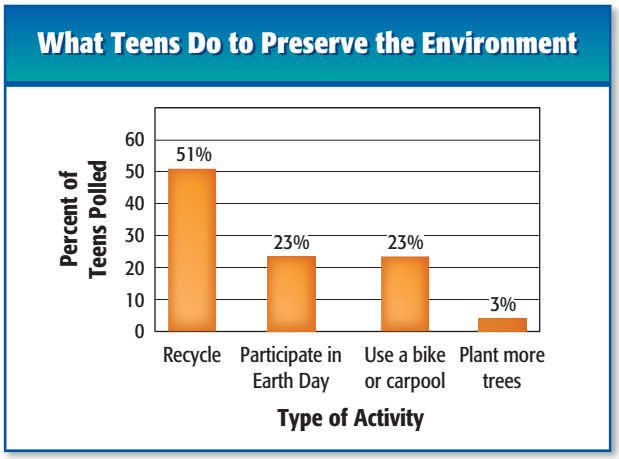
34. **GAMES** A computer game involves a knight on a quest for treasure. At the end of the journey, the knight approaches the two doors shown below.

A servant tells the knight that one of the signs is true and the other is false. Use indirect reasoning to determine which door the knight should choose. Explain your reasoning.

35 **SURVEYS** Luisa's local library conducted an online poll of teens to find out what activities teens participate in to preserve the environment. The results of the poll are shown in the graph.

a. Prove: *More than half of teens polled said that they recycle to preserve the environment.*

b. If 400 teens were polled, verify that 92 said that they participate in Earth Day.



36. **REASONING** James, Hector, and Mandy all have different color cars. Only one of the statements below is true. Use indirect reasoning to determine which statement is true. Explain.

(1) James has a red car.

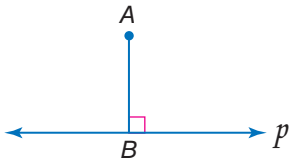
(2) Hector does not have a red car.

(3) Mandy does not have a blue car.

Determine whether each statement about the shortest distance between a point and a line or plane can be proved using a direct or indirect proof. Then write a proof of each statement.

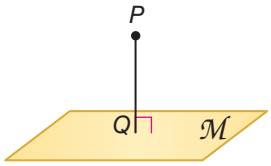
37 **Given:** $\overline{AB} \perp$ line p

Prove: $\overline{AB}$ is the shortest segment from A to line p.



38. **Given:** $\overline{PQ} \perp$ plane M

Prove: $\overline{PQ}$ is the shortest segment from P to plane M.



39. **NUMBER THEORY** In this problem, you will make and prove a conjecture about a number theory relationship.

a. Write an expression for *the sum of the cube of a number and three.*

b. Create a table that includes the value of the expression for 10 different values of n. Include both odd and even values of n.

c. Write a conjecture about n when the value of the expression is even.

d. Write an indirect proof of your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

40. **WRITING IN MATH** Explain the procedure for writing an indirect proof.

41. **OPEN ENDED** Write a statement that can be proven using indirect proof. Include the indirect proof of your statement.

42. **CHALLENGE** If x is a rational number, then it can be represented by the quotient $\frac{a}{b}$ for some integers a and b, if $b \neq 0$. An irrational number cannot be represented by the quotient of two integers. Write an indirect proof to show that the product of a nonzero rational number and an irrational number is an irrational number.

43. CCSS **CRITIQUE** Amber and Raquel are trying to verify the following statement using indirect proof. Is either of them correct? Explain your reasoning.

If the sum of two numbers is even, then the numbers are even.

Amber

The statement is true. If one of the numbers is even and the other number is zero, then the sum is even. Since the hypothesis is true even when the conclusion is false, the statement is true.

Raquel

The statement is true. If the two numbers are odd, then the sum is even. Since the hypothesis is true when the conclusion is false, the statement is true.

44. **WRITING IN MATH** Refer to Exercise 8. Write the contrapositive of the statement and write a direct proof of the contrapositive. How are the direct proof of the contrapositive of the statement and the indirect proof of the statement related?

Standardized Test Practice

45. SHORT RESPONSE Write an equation in slope-intercept form to describe the line that passes through the point (5, 3) and is parallel to the line represented by the equation $-2x + y = -4$.

46. Statement: If $\angle A \cong \angle B$ and $\angle A$ is supplementary to $\angle C$, then $\angle B$ is supplementary to $\angle C$.

Dia is proving the statement above by contradiction. She began by assuming that $\angle B$ is not supplementary to $\angle C$. Which of the following definitions will Dia use to reach a contradiction?

A definition of congruence
B definition of a linear pair
C definition of a right angle
D definition of supplementary angles

47. List the angles of $\triangle MNO$ in order from smallest to largest if $MN = 9$, $NO = 7.5$, and $OM = 12$.

F $\angle N, \angle O, \angle M$
G $\angle O, \angle M, \angle N$
H $\angle O, \angle N, \angle M$
J $\angle M, \angle O, \angle N$

48. SAT/ACT If $b > a$, which of the following must be true?

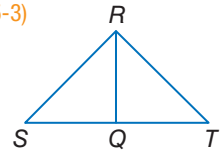
A $-a > -b$
B $3a > b$
C $a^2 < b^2$
D $a^2 < ab$
E $-b > -a$

Spiral Review

49. PROOF Write a two-column proof. (Lesson 5-3)

Given: $\overline{RQ}$ bisects $\angle SRT$.

Prove: $m\angle SQR > m\angle SRQ$

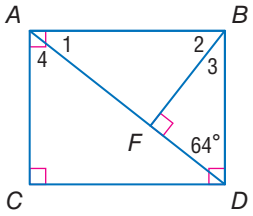


COORDINATE GEOMETRY **Find the coordinates of the circumcenter of each triangle with the given vertices.** (Lesson 5-1)

50. $D(-3, 3), E(3, 2), F(1, -4)$

51. $A(4, 0), B(-2, 4), C(0, 6)$

Find each measure. (Lesson 4-2)



52. $m\angle 1$

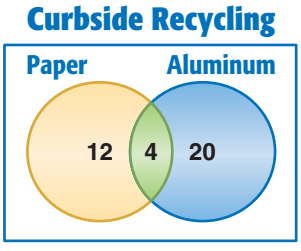
53. $m\angle 4$

COORDINATE GEOMETRY **Find the distance between each pair of parallel lines with the given equations.** (Lesson 3-6)

54. $x + 3y = 6$
$x + 3y = -14$

55. $y = 2x + 2$
$y = 2x - 3$

56. RECYCLING Refer to the Venn diagram that represents the number of neighborhoods in a city with a curbside recycling program for paper or aluminum. (Lesson 2-2)



a. How many neighborhoods recycle aluminum?
b. How many neighborhoods recycle paper or aluminum or both?
c. How many neighborhoods recycle paper and aluminum?

Skills Review

Determine whether each inequality is *true* or *false*.

57. $23 - 11 > 9$

58. $41 - 19 < 21$

59. $57 + 68 < 115$

EXPLORE 5-5

Graphing Technology Lab
The Triangle Inequality

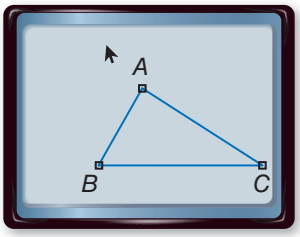
You can use the Cabri™ Jr. application on a TI-83/84 Plus graphing calculator to discover properties of triangles.

Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Activity 1

Construct a triangle. Observe the relationship between the sum of the lengths of two sides and the length of the other side.

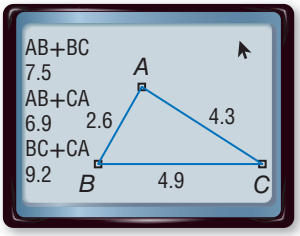
Step 1 Construct a triangle using the triangle tool on the **F2** menu. Then use the **Alph-Num** tool on the **F5** menu to label the vertices as A, B, and C.



Step 1

Step 2 Access the **distance & length** tool, shown as **D. & Length**, under **Measure** on the **F5** menu. Use the tool to measure each side of the triangle.

Step 3 Display $AB + BC$, $AB + CA$, and $BC + CA$ by using the **Calculate** tool on the **F5** menu. Label the measures.



Steps 2 and 3

Step 4 Click and drag the vertices to change the shape of the triangle.

Analyze the Results

1. Replace each ● with <, >, or = to make a true statement.
   $AB + BC$ ● CA $\quad$ $AB + CA$ ● BC $\quad$ $BC + CA$ ● AB

2. Click and drag the vertices to change the shape of the triangle. Then review your answers to Exercise 1. What do you observe?

3. Click on point A and drag it to lie on line BC. What do you observe about AB, BC, and CA? Are A, B, and C the vertices of a triangle? Explain.

4. **Make a conjecture** about the sum of the lengths of two sides of a triangle and the length of the third side.

5. Do the measurements and observations you made in the Activity and in Exercises 1–3 constitute a proof of the conjecture you made in Exercise 4? Explain.

6. Replace each ● with <, >, or = to make a true statement.
   $|AB - BC|$ ● CA $\quad$ $|AB - CA|$ ● BC $\quad$ $|BC - CA|$ ● AB

   Then click and drag the vertices to change the shape of the triangle and review your answers. What do you observe?

7. How could you use your observations to determine the possible lengths of the third side of a triangle if you are given the lengths of the other two sides?

LESSON 5-5

The Triangle Inequality

Then

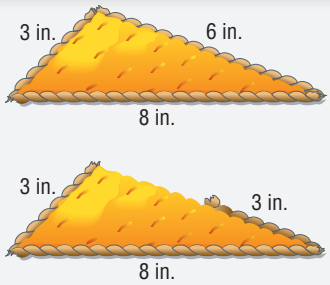
- You recognized and applied properties of inequalities to the relationships between the angles and sides of a triangle.

Now

1. Use the Triangle Inequality Theorem to identify possible triangles.
2. Prove triangle relationships using the Triangle Inequality Theorem.

Why?

- On a home improvement show, a designer wants to use scrap pieces of cording from another sewing project to decorate the triangular throw pillows that she and the homeowner have made. To minimize waste, she wants to use the scraps without cutting them. She selects three scraps at random and tries to form a triangle. Two such attempts are shown.



Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

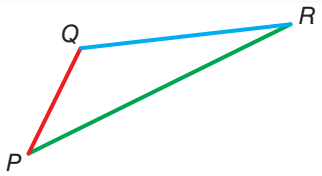
1 The Triangle Inequality

While a triangle is formed by three segments, a special relationship must exist among the lengths of the segments in order for them to form a triangle.

Theorem 5.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Examples $PQ + QR > PR$
$QR + PR > PQ$
$PR + PQ > QR$



You will prove Theorem 5.11 in Exercise 23.

To show that it is not possible to form a triangle with three side lengths, you need only show that one of the three triangle inequalities is not true.

Example 1 Identify Possible Triangles Given Side Lengths

Is it possible to form a triangle with the given side lengths? If not, explain why not.

a. 8 in., 15 in., 17 in.

Check each inequality.

$8 + 15 \stackrel{?}{>} 17$
$23 > 17$ ✓

$8 + 17 \stackrel{?}{>} 15$
$25 > 15$ ✓

$15 + 17 \stackrel{?}{>} 8$
$32 > 8$ ✓

Since the sum of each pair of side lengths is greater than the third side length, sides with lengths 8, 15, and 17 inches will form a triangle.

b. 6 m, 8 m, 14 m

$6 + 8 \stackrel{?}{>} 14$
$14 \not> 14$ ✗

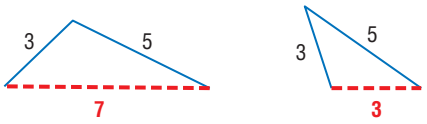
Since the sum of one pair of side lengths is not greater than the third side length, sides with lengths 6, 8, and 14 meters will not form a triangle.

GuidedPractice

1A. 15 yd, 16 yd, 30 yd

1B. 2 ft, 8 ft, 11 ft

When the lengths of two sides of a triangle are known, the third side can be any length in a range of values. You can use the Triangle Inequality Theorem to determine the range of possible lengths for the third side.



Standardized Test Example 2 Find Possible Side Lengths

Test-TakingTip

Testing Choices If you are short on time, you can test each choice to find the correct answer and eliminate any remaining choices.

If the measures of two sides of a triangle are 3 feet and 7 feet, which is the *least* possible whole number measure for the third side?

A 3 ft **B** 4 ft **C** 5 ft **D** 10 ft

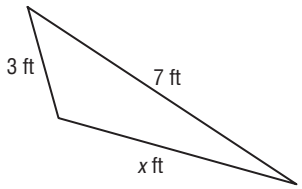
Read the Test Item

You need to determine which value is the least possible measure for the third side of a triangle with sides that measure 3 feet and 7 feet.

Solve the Test Item

To determine the least possible measure from the choices given, first determine the range of possible measures for the third side.

Draw a diagram and let x represent the length of the third side.



Next, set up and solve each of the three triangle inequalities.

$3 + 7 > x$ $\quad$ $3 + x > 7$ $\quad$ $x + 7 > 3$

$10 > x$ or $x < 10$ $\quad$ $x > 4$ $\quad$ $x > -4$

ReadingMath

Multiple Inequality Symbols The compound inequality $4 < x < 10$ is read *x is between 4 and 10.*

Notice that $x > -4$ is always true for any whole number measure for x. Combining the two remaining inequalities, the range of values that fit both inequalities is $x > 4$ and $x < 10$, which can be written as $4 < x < 10$.

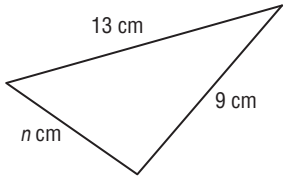
The least whole number value between 4 and 10 is 5. So the correct answer is choice C.

GuidedPractice

2. Which of the following could *not* be the value of n?

F 7 **H** 13

G 10 **J** 22



2 Proofs Using the Triangle Inequality Theorem

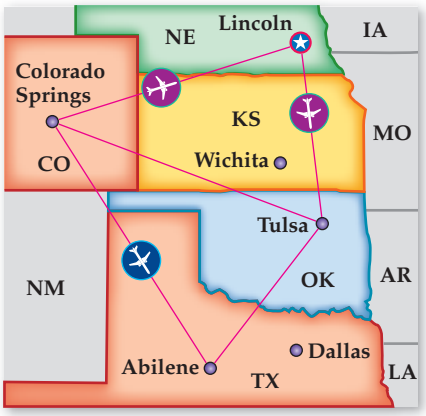
You can use the Triangle Inequality Theorem as a reason in proofs.

Real-WorldLink

A direct flight is not the same as a nonstop flight. For a direct flight, passengers do not change planes, but the plane may make one or more stops before continuing to its final destination.

Real-World Example 3 Proof Using Triangle Inequality Theorem

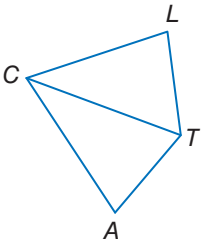
TRAVEL **The distance from Colorado Springs, Springs, Colorado, to Abilene, Texas, is the same as the distance from Colorado Springs to Tulsa, Oklahoma. Prove that a direct flight from Colorado Springs to Tulsa through Lincoln, Nebraska, is a greater distance than a nonstopflight from Colorado Springs to Abilene.**



Draw a simpler diagram of the situation and label the diagram. Draw in side $\overline{LT}$ to form $\triangle CTL$.

Given: $CA = CT$

Prove: $CL + LT > CA$



Proof:

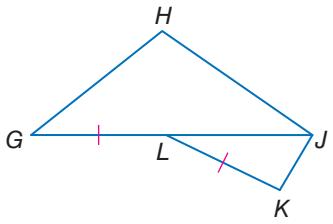
Statements	Reasons
1. $CA = CT$	**1.** Given
2. $CL + LT > CT$	**2.** Triangle Inequality Theorem
3. $CL + LT > CA$	**3.** Substitution

GuidedPractice

3. Write a two-column proof.

Given: $GL = LK$

Prove: $JH + GH > JK$



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Is it possible to form a triangle with the given side lengths? If not, explain why not.**

1. 5 cm, 7 cm, 10 cm **2.** 3 in., 4 in., 8 in. **3.** 6 m, 14 m, 10 m

Example 2

4. MULTIPLE CHOICE If the measures of two sides of a triangle are 5 yards and 9 yards, what is the least possible measure of the third side if the measure is an integer?

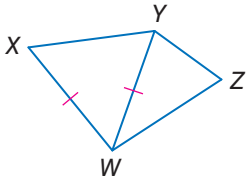
A 4 yd **B** 5 yd **C** 6 yd **D** 14 yd

Example 3

5. PROOF Write a two-column proof.

Given: $\overline{XW} \cong \overline{YW}$

Prove: $YZ + ZW > XW$



Ed Boettcher/CORBIS

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **Is it possible to form a triangle with the given side lengths? If not, explain why not.**

6. 4 ft, 9 ft, 15 ft

7. 11 mm, 21 mm, 16 mm

8. 9.9 cm, 1.1 cm, 8.2 cm

9. 2.1 in., 4.2 in., 7.9 in.

10. $2\frac{1}{2}$ m, $1\frac{3}{4}$ m, $5\frac{1}{8}$ m

11. $1\frac{1}{5}$ km, $4\frac{1}{2}$ km, $3\frac{3}{4}$ km

Example 2 **Find the range for the measure of the third side of a triangle given the measures of two sides.**

12. 4 ft, 8 ft

13. 5 m, 11 m

14. 2.7 cm, 4.2 cm

15. 3.8 in., 9.2 in.

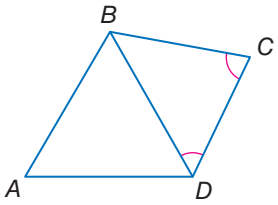
16. $\frac{1}{2}$ km, $3\frac{1}{4}$ km

17. $2\frac{1}{3}$ yd, $7\frac{2}{3}$ yd

Example 3 **PROOF Write a two-column proof.**

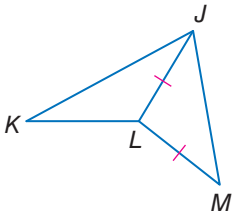
18. **Given:** $\angle BCD \cong \angle CDB$

Prove: $AB + AD > BC$



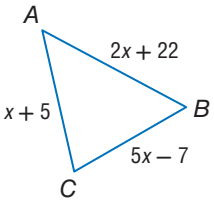
19. **Given:** $\overline{JL} \cong \overline{LM}$

Prove: $KJ + KL > LM$



CCSS **SENSE-MAKING** **Determine the possible values of x.**

20.



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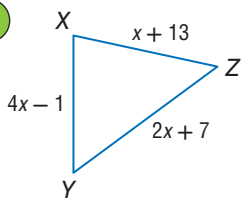
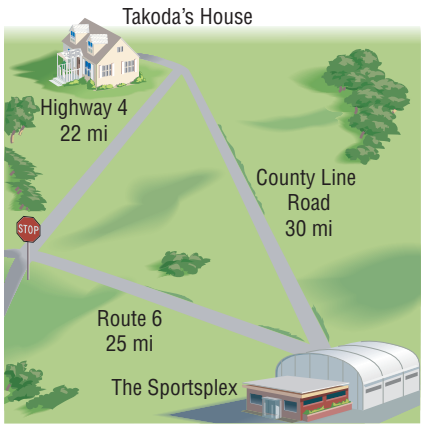


22. **DRIVING** Takoda wants to take the most efficient route from his house to a soccer tournament at The Sportsplex. He can take County Line Road or he can take Highway 4 and then Route 6 to the get to The Sportsplex.

a. Which of the two possible routes is the shortest? Explain your reasoning.

b. Suppose Takoda always drives below the speed limit. If the speed limit on County Line Road is 30 miles per hour and on both Highway 4 and Route 6 it is 55 miles per hour, which route will be faster? Explain.

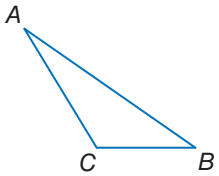


23. **PROOF Write a two-column proof.**

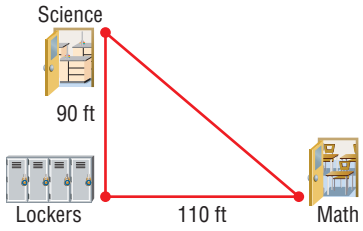
Given: $\triangle ABC$

Prove: $AC + BC > AB$ (Triangle Inequality Theorem)

(*Hint*: Draw auxiliary segment $\overline{CD}$, so that C is between B and D and $\overline{CD} \cong \overline{AC}$.)



24. SCHOOL When Toya goes from science class to math class, she usually stops at her locker. The distance from her science classroom to her locker is 90 feet, and the distance from her locker to her math classroom is 110 feet. What are the possible distances from science class to math class if she takes the hallway that goes directly between the two classrooms?



Find the range of possible measures of x if each set of expressions represents measures of the sides of a triangle.

25. $x, 4, 6$

26. $8, x, 12$

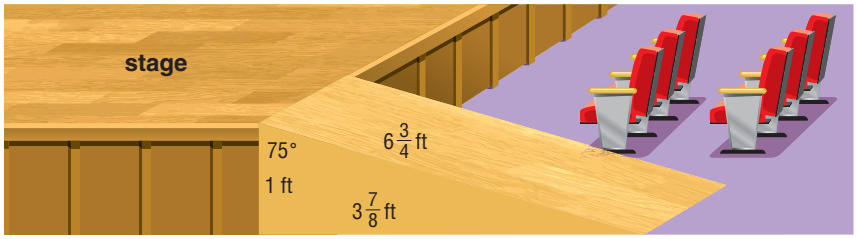
27. $x + 1, 5, 7$

28. $x - 2, 10, 12$

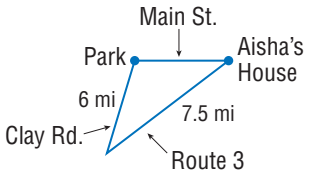
29. $x + 2, x + 4, x + 6$

30. $x, 2x + 1, x + 4$

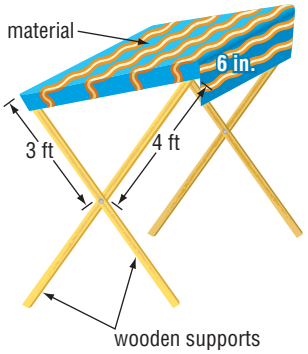
31. DRAMA CLUB Anthony and Catherine are working on a ramp up to the stage for the drama club's next production. Anthony's sketch of the ramp is shown below. Catherine is concerned about the measurements and thinks they should recheck the measures before they start cutting the wood. Is Catherine's concern valid? Explain your reasoning.



32. SENSE-MAKING Aisha is riding her bike to the park and can take one of two routes. The most direct route from her house is to take Main Street, but it is safer to take Route 3 and then turn right on Clay Road as shown. The additional distance she will travel if she takes Route 3 to Clay Road is between how many miles?



33. DESIGN Carlota designed an awning that she and her friends could take to the beach. Carlota decides to cover the top of the awning with material that will drape 6 inches over the front. What length of material should she buy to use with her design so that it covers the top of the awning, including the drape, when the supports are open as far as possible? Assume that the width of the material is sufficient to cover the awning.



ESTIMATION Without using a calculator, determine if it is possible to form a triangle with the given side lengths. Explain.

34. $\sqrt{8}$ ft, $\sqrt{2}$ ft, $\sqrt{35}$ ft

35. $\sqrt{99}$ yd, $\sqrt{48}$ yd, $\sqrt{65}$ yd

36. $\sqrt{3}$ m, $\sqrt{15}$ m, $\sqrt{24}$ m

37. $\sqrt{122}$ in., $\sqrt{5}$ in., $\sqrt{26}$ in.

REASONING **Determine whether the given coordinates are the vertices of a triangle. Explain.**

38. $X(1, -3)$, $Y(6, 1)$, $Z(2, 2)$

39 $F(-4, 3)$, $G(3, -3)$, $H(4, 6)$

40. $J(-7, -1)$, $K(9, -5)$, $L(21, -8)$

41. $Q(2, 6)$, $R(6, 5)$, $S(1, 2)$

42. **MULTIPLE REPRESENTATIONS** In this problem, you will use inequalities to make comparisons between the sides and angles of two triangles.

a. Geometric Draw three pairs of triangles that have two pairs of congruent sides and one pair of sides that is not congruent. Mark each pair of congruent sides. Label each triangle pair ABC and DEF, where $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

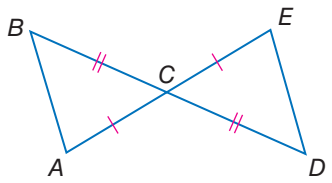
b. Tabular Copy the table below. Measure and record the values of BC, $m\angle A$, EF, and $m\angle D$ for each triangle pair.

Triangle Pair	BC	$m\angle A$	EF	$m\angle D$
1				
2				
3				

c. Verbal Make a conjecture about the relationship between the angles opposite the noncongruent sides of a pair of triangles that have two pairs of congruent legs.

H.O.T. Problems Use Higher-Order Thinking Skills

43. **CHALLENGE** What is the range of possible perimeters for figure $ABCDE$ if $AC = 7$ and $DC = 9$? Explain your reasoning.



44. **REASONING** What is the range of lengths of each leg of an isosceles triangle if the measure of the base is 6 inches? Explain.

45. **WRITING IN MATH** What can you tell about a triangle when given three side lengths? Include at least two items.

46. **CHALLENGE** The sides of an isosceles triangle are whole numbers, and its perimeter is 30 units. What is the probability that the triangle is equilateral?

47. **OPEN ENDED** The length of one side of a triangle is 2 inches. Draw a triangle in which the 2-inch side is the shortest side and one in which the 2-inch side is the longest side. Include side and angle measures on your drawing.

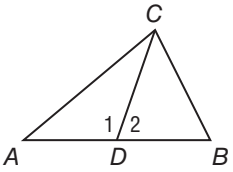
48. **WRITING IN MATH** Suppose your house is $\frac{3}{4}$ mile from a park and the park is 1.5 miles from a shopping center.

a. If your house, the park, and the shopping center are noncollinear, what do you know about the distance from your house to the shopping center? Explain your reasoning.

b. If the three locations are collinear, what do you know about the distance from your house to the shopping center? Explain your reasoning.

Standardized Test Practice

49. If $\overline{DC}$ is a median of $\triangle ABC$ and $m\angle 1 > m\angle 2$, which of the following statements is not true?



A $AD = BD$ **C** $AC > BC$

B $m\angle ADC = m\angle BDC$ **D** $m\angle 1 > m\angle B$

50. SHORT RESPONSE A high school soccer team has a goal of winning at least 75% of their 15 games this season. In the first three weeks, the team has won 5 games. How many more games must the team win to meet their goal?

51. Which of the following is a logical conclusion based on the statement and its converse below?

Statement: If a polygon is a rectangle, then it has four sides.

Converse: If a polygon has four sides, then it is a rectangle.

F The statement and its converse are both true.

G The statement is false; the converse is false.

H The statement is true; the converse is false.

J The statement is false; the converse is true.

52. SAT/ACT When 7 is subtracted from $14w$, the result is z. Which of the following equations represents this statement?

A $7 - 14w = z$ **D** $z = 14w - 7$

B $z = 14w + 7$ **E** $7 + 14w = 7z$

C $7 - z = 14w$

Spiral Review

State the assumption you would make to start an indirect proof of each statement. (Lesson 5-4)

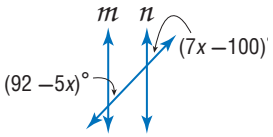
53. If $4y + 17 = 41$, then $y = 6$.

54. If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the two lines are parallel.

55. GEOGRAPHY The distance between San Jose, California, and Las Vegas, Nevada, is about 375 miles. The distance from Las Vegas to Carlsbad, California, is about 243 miles. Use the Triangle Inequality Theorem to find the possible distance between San Jose and Carlsbad. (Lesson 5-3)

Find x so that $m \parallel n$. Identify the postulate or theorem you used. (Lesson 3-5)

56.



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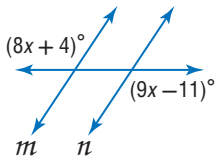

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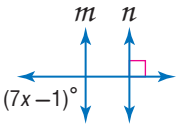


ALGEBRA **Find x and JK if J is between K and L.** (Lesson 1-2)

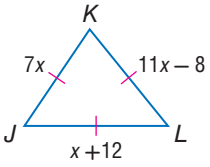
59. $KJ = 3x$, $JL = 6x$, and $KL = 12$

60. $KJ = 3x - 6$, $JL = x + 6$, and $KL = 24$

Skills Review

Find x and the measures of the unknown sides of each triangle.

61.



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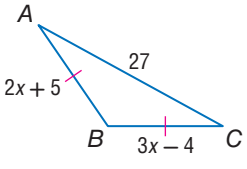

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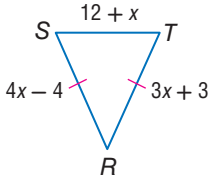


LESSON 5-6

Inequalities in Two Triangles

Then

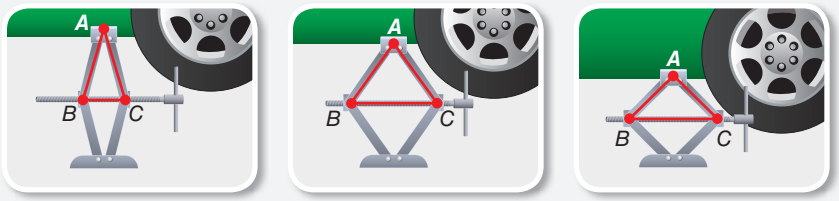
- You used inequalities to make comparisons in one triangle.

Now

1. Apply the Hinge Theorem or its converse to make comparisons in two triangles.
2. Prove triangle relationships using the Hinge Theorem or its converse.

Why?

- A car jack is used to lift a car. The jack shown below is one of the simplest still in use today. Notice that as the jack is lowered, the legs of isosceles $\triangle ABC$ remain congruent, but the included angle A widens and $\overline{BC}$, the side opposite $\angle A$, lengthens.



Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

1 Make sense of problems and persevere in solving them.

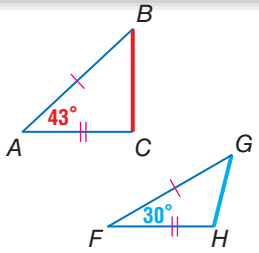
1 Hinge Theorem

The observation in the example above is true of any type of triangle and illustrates the following theorems.

Theorems Inequalities in Two Triangles

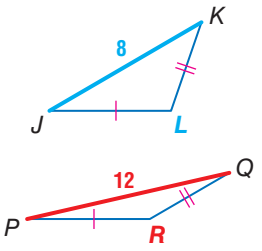
5.13 Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

Example: If $\overline{AB} \cong \overline{FG}$, $\overline{AC} \cong \overline{FH}$, and $m\angle A > m\angle F$, then $BC > GH$.



5.14 Converse of the Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the third side in the first is longer than the third side in the second triangle, then the included angle measure of the first triangle is greater than the included angle measure in the second triangle.

Example: If $\overline{JL} \cong \overline{PR}$, $\overline{KL} \cong \overline{QR}$, and $PQ > JK$, then $m\angle R > m\angle L$.

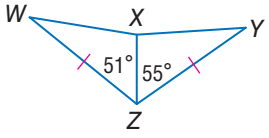


The proof of Theorem 5.13 is on p. 372. You will prove Theorem 5.14 in Exercise 28.

Example 1 Use the Hinge Theorem and its Converse

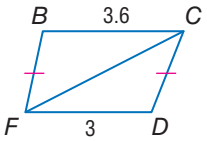
Compare the given measures.

a. WX and XY



In $\triangle WXZ$ and $\triangle YXZ$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ} \cong \overline{XZ}$, and $\angle YZX > \angle WZX$. By the Hinge Theorem, $m\angle WZX < m\angle YZX$, so $WX < XY$.

b. $m\angle FCD$ and $m\angle BFC$

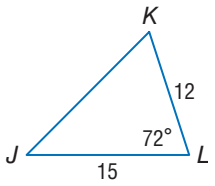


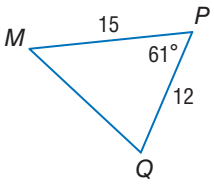
In $\triangle BCF$ and $\triangle DFC$, $\overline{BF} \cong \overline{DC}$, $\overline{FC} \cong \overline{CF}$, and $BC > FD$. By the Converse of the Hinge Theorem, $\angle BFC > \angle DCF$.

GuidedPractice

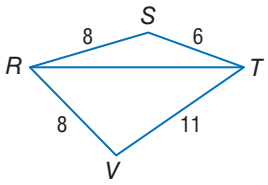
Compare the given measures.

1A. JK and MQ





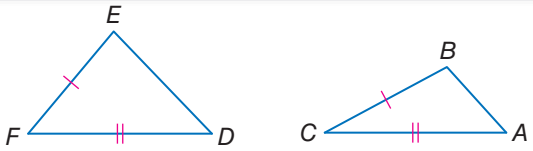
1B. $m\angle SRT$ and $m\angle VRT$



Proof Hinge Theorem

Given: $\triangle ABC$ and $\triangle DEF$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$, $m\angle F > m\angle C$

Prove: $DE > AB$



Proof:

We are given that $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$. We also know that $m\angle F > m\angle C$.

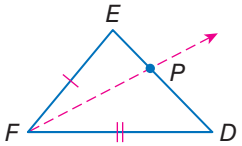
Draw auxiliary ray FP such that $m\angle DFP = m\angle C$ and that $\overline{PF} \cong \overline{BC}$. This leads to two cases.

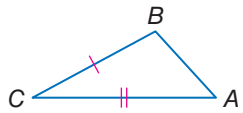
StudyTip

SAS and SSS Inequality Theorem The Hinge Theorem is also called the SAS Inequality Theorem. The Converse of the Hinge Theorem is also called the SSS Inequality Theorem.

Case 1 P lies on $\overline{DE}$.

Then $\triangle FPD \cong \triangle CBA$ by SAS. Thus, $PD = BA$ by CPCTC and the definition of congruent segments.

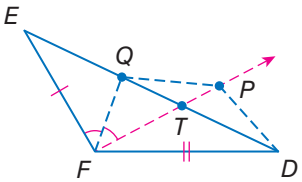


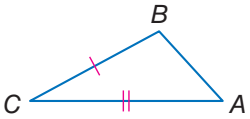


By the Segment Addition Postulate, $DE = EP + PD$. Also, $DE > PD$ by the definition of inequality. Therefore, $DE > AB$ by substitution.

Case 2 P does not lie on $\overline{DE}$.

Then let the intersection of $\overline{FP}$ and $\overline{ED}$ be point T, and draw another auxiliary segment $\overline{FQ}$ such that Q is on $\overline{DE}$ and $\angle EFQ \cong \angle QFP$. Then draw auxiliary segments $\overline{PD}$ and $\overline{PQ}$.





Since $\overline{FP} \cong \overline{BC}$ and $\overline{BC} \cong \overline{EF}$, we have $\overline{FP} \cong \overline{EF}$ by the Transitive Property. Also $\overline{QF}$ is congruent to itself by the Reflexive Property. Thus, $\triangle EFQ \cong \triangle PFQ$ by SAS. By CPCTC, $\overline{EQ} \cong \overline{PQ}$ or $EQ = PQ$. Also, $\triangle FPD \cong \triangle CBA$ by SAS. So, $\overline{PD} \cong \overline{BA}$ by CPCTC and $PD = BA$.

In $\triangle QPD$, $QD + PQ > PD$ by the Triangle Inequality Theorem. By substitution, $QD + EQ >$ PD. Since $ED = QD + EQ$ by the Segment Addition Postulate, $ED > PD$. Using substitution, $ED > BA$ or $DE > AB$.

Real-World Link

There are over 225,000 miles of groomed and marked snowmobile trails in North America.

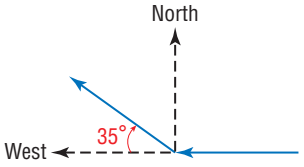
Source: International Snowmobile Manufacturers Association

You can use the Hinge Theorem to solve real-world problems.

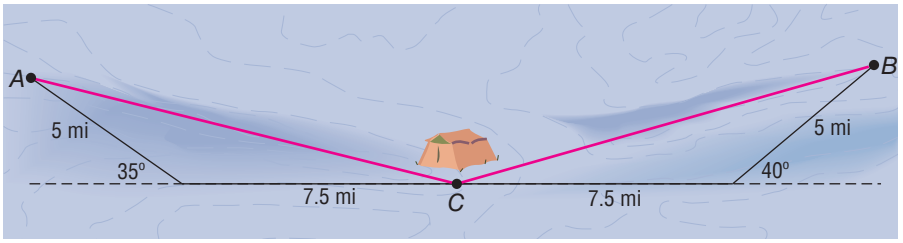
Real-World Example 2 Use the Hinge Theorem

SNOWMOBILING **Two groups of snowmobilers leave from the same base camp. Group A goes 7.5 miles due west and then turns 35° north of west and goes 5 miles. Group B goes 7.5 miles due east and then turns 40° north of east and goes 5 miles. At this point, which group is farther from the base camp? Explain your reasoning.**

Understand Using the sets of directions given in the problem, you need to determine which snowmobile group is farther from the base camp. A turn of 35° north of west is correctly interpreted as shown.



Plan Draw a diagram of the situation.



Problem-Solving Tip

Draw a Diagram Draw a diagram to help you see and correctly interpret a problem that has been described in words.

The paths taken by each group and the straight-line distance back to the camp form two triangles. Each group goes 7.5 miles and then turns and goes 5 miles.

Use linear pairs to find the measures of the included angles. Then apply the Hinge Theorem to compare the distance each group is from base camp.

Solve The included angle for the path made by Group A measures $180 - 35$ or 145. The included angle for the path made by Group B is $180 - 40$ or 140.

Since $145 > 140$, $AC > BC$ by the Hinge Theorem. So Group A is farther from the base camp.

Check Group B turned 5° more than Group A did back toward base camp, so they should be closer to base camp than Group A. Thus, Group A should be farther from the base camp. ✔

Guided Practice

2A. SKIING Two groups of skiers leave from the same lodge. Group A goes 4 miles due east and then turns 70° north of east and goes 3 miles. Group B goes 4 miles due west and then turns 75° north of west and goes 3 miles. At this point, which group is *farther* from the lodge? Explain your reasoning.

2B. SKIING In problem 2A, suppose Group A instead went 4 miles west and then turned 45° north of west and traveled 3 miles. Which group would be *closer* to the lodge? Explain your reasoning.

When the included angle of one triangle is greater than the included angle in a second triangle, the Converse of the Hinge Theorem is used.

StudyTip

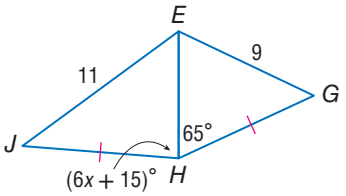
Using Additional Facts

When finding a range for the possible values for x, you may need to use one of the following facts.

- The measure of any angle is always greater than 0 and less than 180.
- The measure of any segment is always greater than 0.

Example 3 Apply Algebra to the Relationships in Triangles

ALGEBRA **Find the range of possible values for x.**



Step 1 From the diagram, we know that $\overline{JH} \cong \overline{GH}$, $\overline{EH} \cong \overline{EH}$, and $JE > EG$.

$m\angle JHE > m\angle EHG$ Converse of the Hinge Theorem

$6x + 15 > 65$ Substitution

$x > 8\frac{1}{3}$ Solve for x.

Step 2 Use the fact that the measure of any angle in a triangle is less than 180 to write a second inequality.

$m\angle JHE < 180$

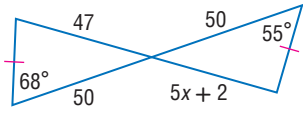
$6x + 15 < 180$ Substitution

$x < 27.5$ Solve for x.

Step 3 Write $x > 8\frac{1}{3}$ and $x < 27.5$ as the compound inequality $8\frac{1}{3} < x < 27.5$.

GuidedPractice

3. Find the range of possible values for x.



2 Prove Relationships In Two Triangles

You can use the Hinge Theorem and its converse to prove relationships in two triangles.

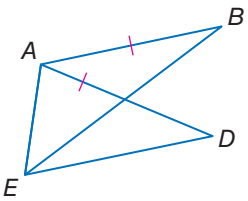
Example 4 Prove Triangle Relationships Using Hinge Theorem

Write a two-column proof.

Given: $\overline{AB} \cong \overline{AD}$

Prove: $EB > ED$

Proof:



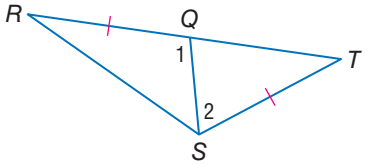
Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. $\overline{AE} \cong \overline{AE}$	2. Reflexive Property
3. $m\angle EAB = m\angle EAD + m\angle DAB$	3. Angle Addition Postulate
4. $m\angle EAB > m\angle EAD$	4. Definition of Inequality
5. $EB > ED$	5. Hinge Theorem

GuidedPractice

4. Write a two-column proof.

Given: $\overline{RQ} \cong \overline{ST}$

Prove: $RS > TQ$

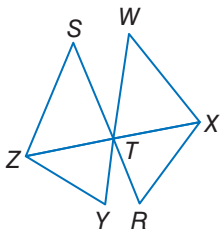


Example 5 Prove Relationships Using Converse of Hinge Theorem

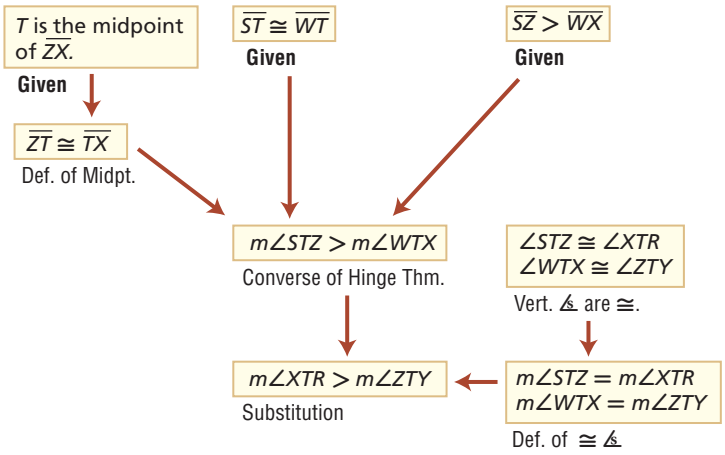
Write a flow proof.

Given: T is the midpoint of $\overline{ZX}$.
$\overline{ST} \cong \overline{WT}$
$SZ > WX$

Prove: $m\angle XTR > m\angle ZTY$



Flow Proof:

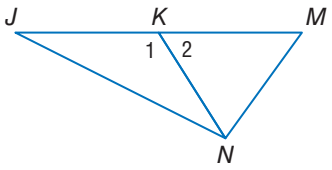


GuidedPractice

5. Write a two-column proof.

Given: $\overline{NK}$ is a median of $\triangle JMN$.
$JN > NM$

Prove: $m\angle 1 > m\angle 2$

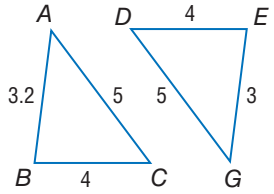


Check Your Understanding

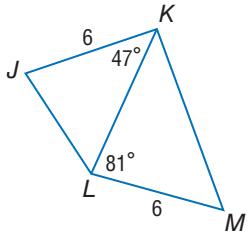
= Step-by-Step Solutions begin on page R14.

Example 1 **Compare the given measures.**

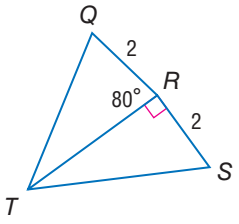
1. $m\angle ACB$ and $m\angle GDE$



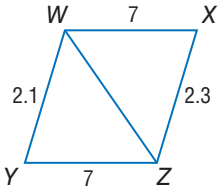
2. JL and KM



3 QT and ST



4. $m\angle XWZ$ and $m\angle YZW$

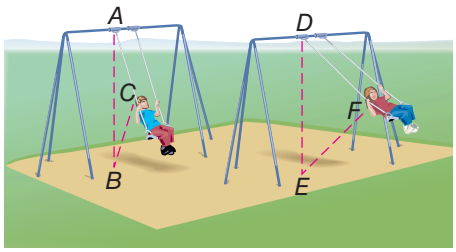


Example 2

5. SWINGS The position of the swing changes based on how hard the swing is pushed.

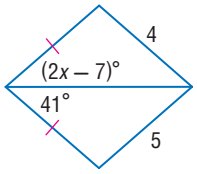
a. Which pairs of segments are congruent?

b. Is the measure of $\angle A$ or the measure of $\angle D$ greater? Explain.

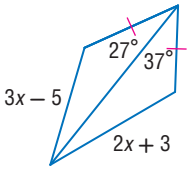


Example 3 **Find the range of possible values for x.**

6.

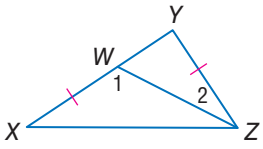


7.

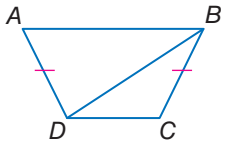


Examples 4–5 CCSS **ARGUMENTS** **Write a two-column proof.**

8. Given: $\triangle YZX$
$\overline{YZ} \cong \overline{XW}$
Prove: $ZX > YW$



9. Given: $\overline{AD} \cong \overline{CB}$
$DC < AB$
Prove: $m\angle CBD < m\angle ADB$

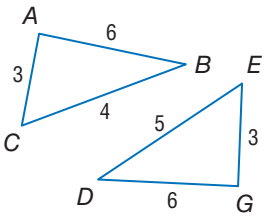


Practice and Problem Solving

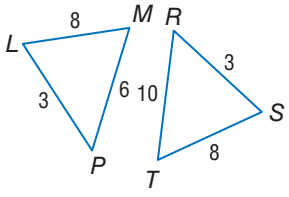
Extra Practice is on page R5.

Example 1 **Compare the given measures.**

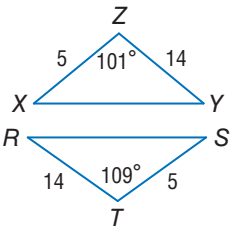
10. $m\angle BAC$ and $m\angle DGE$



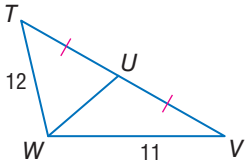
11. $m\angle MLP$ and $m\angle TSR$



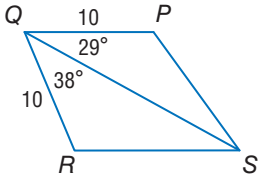
12. SR and XY



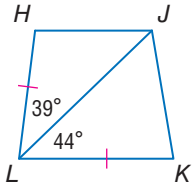
13. $m\angle TUW$ and $m\angle VUW$



14. PS and SR



15. JK and HJ



Example 2

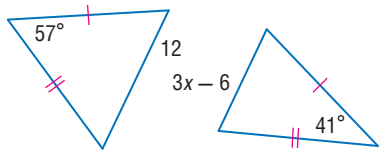
16. CAMPING Pedro and Joel are camping in a national park. One morning, Pedro decides to hike to the waterfall. He leaves camp and goes 5 miles east then turns 15° south of east and goes 2 more miles. Joel leaves the camp and travels 5 miles west, then turns 35° north of west and goes 2 miles to the lake for a swim.

a. When they reach their destinations, who is closer to the camp? Explain your reasoning. Include a diagram.

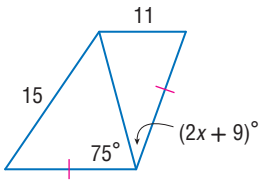
b. Suppose instead of turning 35° north of west, Joel turned 10° south of west. Who would then be farther from the camp? Explain your reasoning. Include a diagram.

Example 3 Find the range of possible values for x.

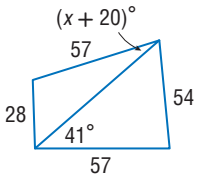
17.



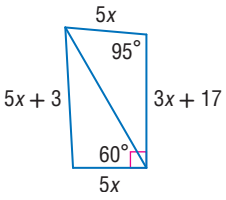
18.



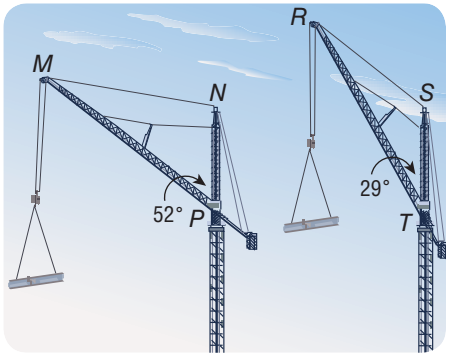
19



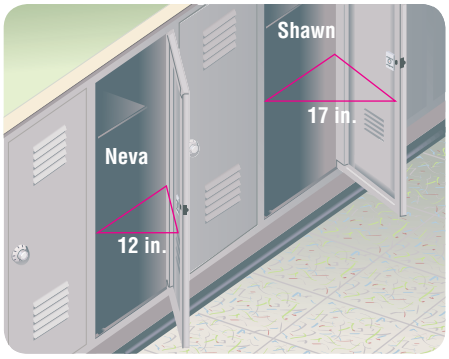
20.



21. CRANES In the diagram, a crane is shown lifting an object to two different heights. The length of the crane's arm is fixed, and $\overline{MP} \cong \overline{RT}$. Is $\overline{MN}$ or $\overline{RS}$ shorter? Explain your reasoning.



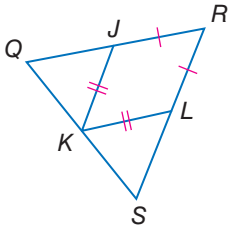
22. LOCKERS Neva and Shawn both have their lockers open as shown in the diagram. Whose locker forms a larger angle? Explain your reasoning.



Examples 4–5 CCSS **ARGUMENTS** Write a two-column proof.

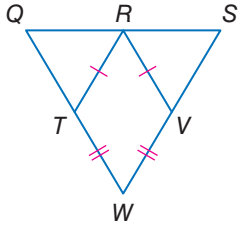
23. **Given:** $\overline{LK} \cong \overline{JK}$, $\overline{RL} \cong \overline{RJ}$
K is the midpoint of $\overline{QS}$.
$m\angle SKL > m\angle QKJ$

Prove: $RS > QR$



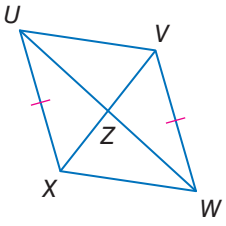
24. **Given:** $\overline{VR} \cong \overline{RT}$, $\overline{WV} \cong \overline{WT}$
$m\angle SRV > m\angle QRT$
R is the midpoint of $\overline{SQ}$.

Prove: $WS > WQ$



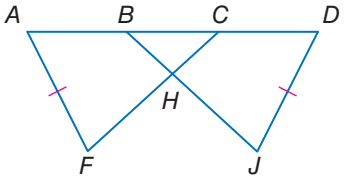
25. **Given:** $\overline{XU} \cong \overline{VW}$, $VW > XW$
$\overline{XU} \parallel \overline{VW}$

Prove: $m\angle XZU > m\angle UZV$

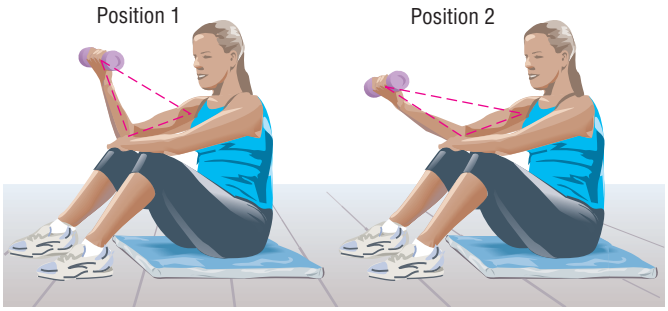


26. **Given:** $\overline{AF} \cong \overline{DJ}$, $\overline{FC} \cong \overline{JB}$
$AB > DC$

Prove: $m\angle AFC > m\angle DJB$



27 **EXERCISE** Anica is doing knee-supported bicep curls as part of her strength training.



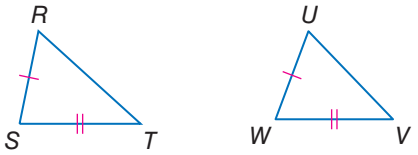
a. Is the distance from Anica's fist to her shoulder greater in Position 1 or Position 2? Justify your answer using measurement.

b. Is the measure of the angle formed by Anica's elbow greater in Position 1 or Position 2? Explain your reasoning.

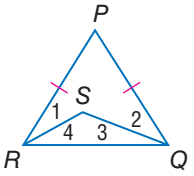
28. **PROOF** Use an indirect proof to prove the SSS Inequality Theorem (Theorem 5.14).

Given: $\overline{RS} \cong \overline{UW}$
$\overline{ST} \cong \overline{WV}$
$RT > UV$

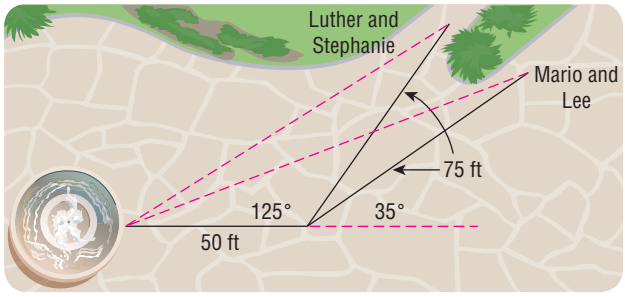
Prove: $m\angle S > m\angle W$



29. **PROOF** If $\overline{PR} \cong \overline{PQ}$ and $SQ > SR$, write a two-column proof to prove $m\angle 1 < m\angle 2$.



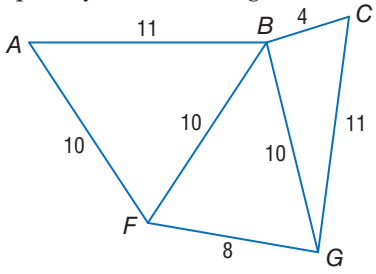
30. **SCAVENGER HUNT** Stephanie, Mario, Lee, and Luther are participating in a scavenger hunt as part of a geography lesson. Their map shows that the next clue is 50 feet due east and then 75 feet 35° east of north starting from the fountain in the school courtyard. When they get ready to turn and go 75 feet 35° east of north, they disagree about which way to go, so they split up and take the paths shown in the diagram below.



a. Which pair chose the correct path? Explain your reasoning.

b. Which pair is closest to the fountain when they stop? Explain your reasoning.

CCSS **SENSE-MAKING** **Use the figure at the right to write an inequality relating the given pair of angle or segment measures.**

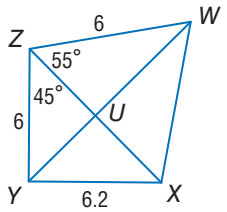


31. CB and AB

32. $m\angle FBG$ and $m\angle BFA$

33. $m\angle BGC$ and $m\angle FBA$

Use the figure at the right to write an inequality relating the given pair of angles or segment measures.



34. $m\angle ZUY$ and $m\angle ZUW$

35 WU and YU

36. WX and XY

37. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate properties of polygons.

a. Geometric Draw a three-sided, a four-sided, and a five-sided polygon. Label the 3-sided polygon ABC, the four-sided polygon $FGHJ$, and the five-sided polygon $PQRST$. Use a protractor to measure and label each angle.

b. Tabular Copy and complete the table below.

Number of sides	Angle Measures				Sum of Angles
3	$m\angle A$		$m\angle C$		
	$m\angle B$				
4	$m\angle F$		$m\angle H$		
	$m\angle G$		$m\angle J$		
5	$m\angle P$		$m\angle S$		
	$m\angle Q$		$m\angle T$		
	$m\angle R$				

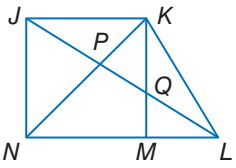
c. Verbal Make a conjecture about the relationship between the number of sides of a polygon and the sum of the measures of the angles of the polygon.

d. Logical What type of reasoning did you use in part **c**? Explain.

e. Algebraic Write an algebraic expression for the sum of the measures of the angles for a polygon with n sides.

H.O.T. Problems Use Higher-Order Thinking Skills

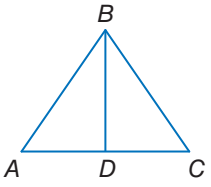
38. **CHALLENGE** If $m\angle LJN > m\angle KJL$, $KJ \cong JN$, and $JN \perp NL$, which is greater, $m\angle LKN$ or $m\angle LNK$? Explain your reasoning.



39. **OPEN ENDED** Give a real-world example of an object that uses a hinge. Draw two sketches in which the hinge on your object is adjusted to two different positions. Use your sketches to explain why Theorem 5.13 is called the Hinge Theorem.

40. **CHALLENGE** Given $\triangle RST$ with median $\overline{RQ}$, if RT is greater than or equal to RS, what are the possible classifications of $\triangle RQT$? Explain your reasoning.

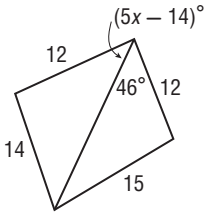
41. **CCSS PRECISION** If $\overline{BD}$ is a median and $AB < BC$, then $\angle BDC$ is *always*, *sometimes*, or *never* an acute angle. Explain.



42. **WRITING IN MATH** Compare and contrast the Hinge Theorem to the SAS Postulate for triangle congruence.

Standardized Test Practice

43. SHORT RESPONSE Write an inequality to describe the possible range of values for x.



44. Which of the following is the inverse of the statement *If it is snowing, then Steve wears his snow boots?*

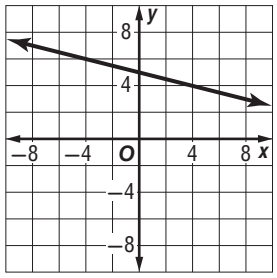
A If Steve wears his snow boots, then it is snowing.

B If it is not snowing, then Steve does not wear his snow boots.

C If it is not snowing, then Steve wears his snow boots.

D If it never snows, then Steve does not own snow boots.

45. ALGEBRA Which linear function best describes the graph shown?



F $y = -\frac{1}{4}x + 5$

G $y = -\frac{1}{4}x - 5$

H $y = \frac{1}{4}x + 5$

J $y = \frac{1}{4}x - 5$

46. SAT/ACT If the side of a square is $x + 3$, then the diagonal of the square is

A $x^2 + 1$

B $x\sqrt{2} + 3\sqrt{2}$

C $2x + 6$

D $x^2\sqrt{2} + 6$

E $x^2 + 9$

Spiral Review

Find the range for the measure of the third side of a triangle given the measures of two sides. (Lesson 5-5)

47. 3.2 cm, 4.4 cm

48. 5 ft, 10 ft

49. 3 m, 9 m

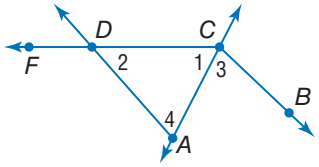
50. CRUISES Ally asked Tavia the cost of a cruise she and her best friend went on after graduation. Tavia could not remember how much it cost per person, but she did remember that the total cost was over $500. Use indirect reasoning to show that the cost for one person was more than $250. (Lesson 5-4)

Draw and label a figure to represent the congruent triangles. Then find x. (Lesson 4-3)

51. $\triangle QRS \cong \triangle GHJ$, $RS = 12$, $QR = 10$, $QS = 6$, and $HJ = 2x - 4$.

52. $\triangle ABC \cong \triangle XYZ$, $AB = 13$, $AC = 19$, $BC = 21$, and $XY = 3x + 7$.

Use the figure at the right. (Lesson 1-4)



53. Name the vertex of $\angle 4$.

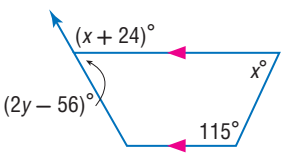
54. What is another name for $\angle 2$?

55. What is another name for $\angle BCA$?

Skills Review

Find the value of the variable(s) in each figure. Explain your reasoning.

56.



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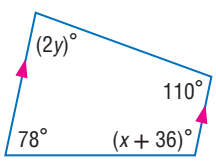

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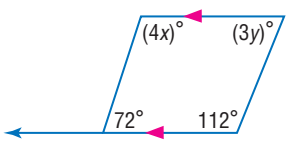


CHAPTER 5

Study Guide and Review

Study Guide

KeyConcepts

Special Segments in Triangles (Lessons 5-1 and 5-2)

- The special segments of triangles are perpendicular bisectors, angle bisectors, medians, and altitudes.
- The intersection points of each of the special segments of a triangle are called the points of concurrency.
- The points of concurrency for a triangle are the circumcenter, incenter, centroid, and orthocenter.

Indirect Proof (Lesson 5-4)

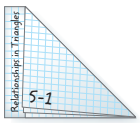
- Writing an Indirect Proof:
  1. Assume that the conclusion is false.
  2. Show that this assumption leads to a contradiction.
  3. Since the false conclusion leads to an incorrect statement, the original conclusion must be true.

Triangle Inequalities (Lessons 5-3, 5-5, and 5-6)

- The largest angle in a triangle is opposite the longest side, and the smallest angle is opposite the shortest side.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- **SAS Inequality** (Hinge Theorem): In two triangles, if two sides are congruent, then the measure of the included angle determines which triangle has the longer third side.
- **SSS Inequality**: In two triangles, if two corresponding sides of each triangle are congruent, then the length of the third side determines which triangle has the included angle with the greater measure.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

altitude (p. 337)

centroid (p. 335)

circumcenter (p. 325)

concurrent lines (p. 325)

incenter (p. 328)

indirect proof (p. 355)

indirect reasoning (p. 355)

median (p. 335)

orthocenter (p. 337)

perpendicular bisector (p. 324)

point of concurrency (p. 325)

proof by contradiction (p. 355)

VocabularyCheck

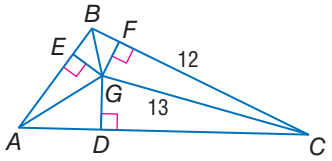
State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The altitudes of a triangle intersect at the centroid.
2. The point of concurrency of the medians of a triangle is called the incenter.
3. The point of concurrency is the point at which three or more lines intersect.
4. The circumcenter of a triangle is equidistant from the vertices of the triangle.
5. To find the centroid of a triangle, first construct the angle bisectors.
6. The perpendicular bisectors of a triangle are concurrent lines.
7. To start a proof by contradiction, first assume that what you are trying to prove is true.
8. A proof by contradiction uses indirect reasoning.
9. A median of a triangle connects the midpoint of one side of the triangle to the midpoint of another side of the triangle.
10. The incenter is the point at which the angle bisectors of a triangle intersect.

Lesson-by-Lesson Review

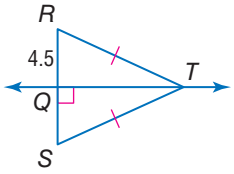
5-1 Bisectors of Triangles

11. Find EG if G is the incenter of $\triangle ABC$.

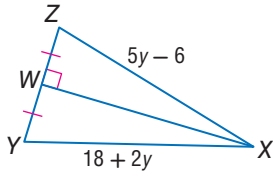


Find each measure.

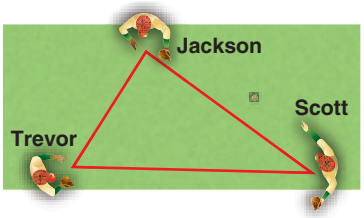
12. RS



13. XZ

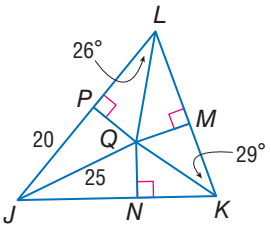


14. BASEBALL Jackson, Trevor, and Scott are warming up before a baseball game. One of their warm-up drills requires three players to form a triangle, with one player in the middle. Where should the fourth player stand so that he is the same distance from the other three players?



Example 1

Find each measure if Q is the incenter of $\triangle JKL$.



a. $\angle QJK$

$m\angle KLP + m\angle MKN + m\angle NJP = 180$	△ Sum Theorem
$2(26) + 2(29) + m\angle NJP = 180$	Substitution
$110 + m\angle NJP = 180$	Simplify.
$m\angle NJP = 70$	Subtract.

Since $\overrightarrow{JQ}$ bisects $\angle NJP$, $2m\angle QJK = m\angle NJP$.
So, $m\angle QJK = \frac{1}{2}m\angle NJP$, so $m\angle QJK = \frac{1}{2}(70)$ or 35.

b. QP

$a^2 + b^2 = c^2$	Pythagorean Theorem
$(QP)^2 + 20^2 = 25^2$	Substitution
$(QP)^2 + 400 = 625$	$20^2 = 400$ and $25^2 = 625$
$(QP)^2 = 225$	Subtract.
$QP = 15$	Simplify.

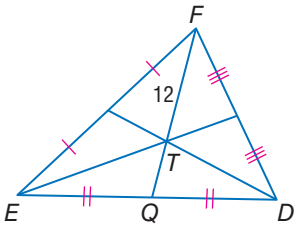
5-2 Medians and Altitudes of Triangles

15. The vertices of $\triangle DEF$ are $D(0, 0)$, $E(0, 7)$, and $F(6, 3)$. Find the coordinates of the orthocenter of $\triangle DEF$.

16. PROM Georgia is on the prom committee. She wants to hang a dozen congruent triangles from the ceiling so that they are parallel to the floor. She sketched out one triangle on a coordinate plane with coordinates (0, 4), (3, 8), and (6, 0). If each triangle is to be hung by one chain, what are the coordinates of the point where the chain should attach to the triangle?

Example 2

In $\triangle EDF$, T is the centroid and $FT = 12$. Find TQ.

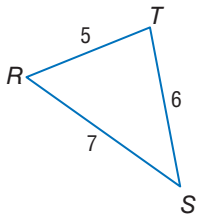


$FT = \frac{2}{3}FQ$	
$FT = \frac{2}{3}(FT + TQ)$	
$12 = \frac{2}{3}(12 + TQ)$	$FT = 12$
$12 = 8 + \frac{2}{3}TQ$	Distributive Property
$4 = \frac{2}{3}TQ$	Subtract.
$6 = TQ$	Multiply.

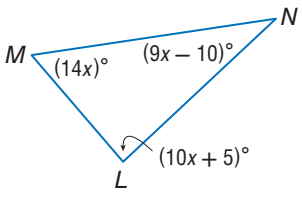
5-3 Inequalities in One Triangle

List the angles and sides of each triangle in order from smallest to largest.

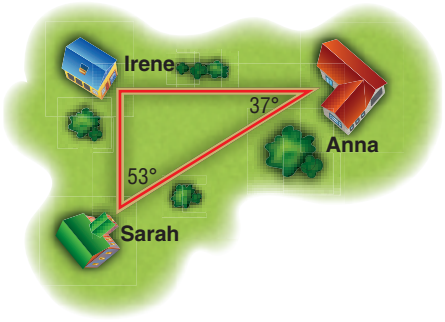
17.



18.

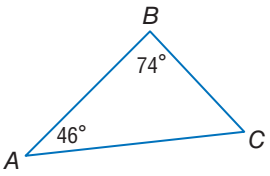


19. **NEIGHBORHOODS** Anna, Sarah, and Irene live at the intersections of the three roads that make the triangle shown. If the girls want to spend the afternoon together, is it a shorter path for Anna to stop and get Sarah and go onto Irene's house, or for Sarah to stop and get Irene and then go on to Anna's house?



Example 3

List the angles and sides of $\triangle ABC$ in order from smallest to largest.



a. First, find the missing angle measure using the Triangle Sum Theorem.

$m\angle C = 180 - (46 + 74)$ or 60

So, the angles from smallest to largest are $\angle A$, $\angle C$, and $\angle B$.

b. The sides from shortest to longest are $\overline{BC}$, $\overline{AB}$, and $\overline{AC}$.

5-4 Indirect Proof

State the assumption you would make to start an indirect proof of each statement.

20. $m\angle A \geq m\angle B$

21. $\triangle FGH \cong \triangle MNO$

22. $\triangle KLM$ is a right triangle.

23. If $3y < 12$, then $y < 4$.

24. Write an indirect proof to show that if two angles are complementary, neither angle is a right angle.

25. **MOVIES** Isaac bought two DVD's and spent over $50. Use indirect reasoning to show that at least one of the DVD's he purchased was over $25.

Example 4

State the assumption necessary to start an indirect proof of each statement.

a. $\overline{XY} \ncong \overline{JK}$

$\overline{XY} \cong \overline{JK}$

b. If $3x < 18$, then $x < 6$.

The conclusion of the conditional statement is $x < 6$. The negation of the conclusion is $x \geq 6$.

c. $\angle 2$ is an acute angle.

If $\angle 2$ *is an acute angle* is false, then $\angle 2$ *is not an acute angle* must be true. This means that $\angle 2$ *is an obtuse or right angle* must be true.

5-5 The Triangle Inequality

Is it possible to form a triangle with the given lengths? If not, explain why not.

26. 5, 6, 9

27. 3, 4, 8

Find the range for the measure of the third side of a triangle given the measure of two sides.

28. 5 ft, 7 ft

29. 10.5 cm, 4 cm

30. **BIKES** Leonard rides his bike to visit Josh. Since High Street is closed, he has to travel 2 miles down Main Street and turn to travel 3 miles farther on 5th Street. If the three streets form a triangle with Leonard and Josh's house as two of the vertices, find the range of the possible distance between Leonard and Josh's houses when traveling straight down High Street.

Example 5

Is it possible to form a triangle with the lengths 7, 10, and 9 feet? If not, explain why not.

Check each inequality.

$7 + 10 > 9$ $\quad$ $7 + 9 > 10$ $\quad$ $10 + 9 > 7$

$17 > 9$ ✓ $\quad$ $16 > 10$ ✓ $\quad$ $19 > 7$ ✓

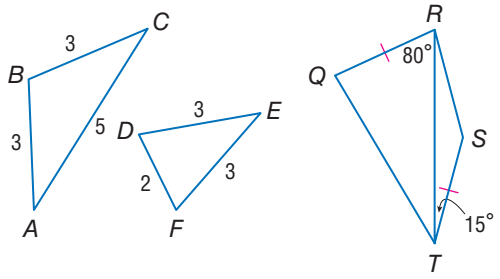
Since the sum of each pair of side lengths is greater than the third side length, sides with lengths 7, 10, and 9 feet will form a triangle.

5-6 Inequalities in Two Triangles

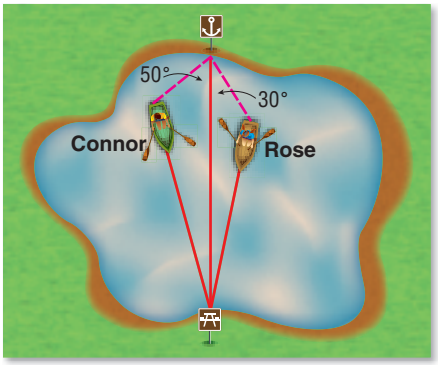
Compare the given measures.

31. $m\angle ABC$, $m\angle DEF$

32. QT and RS

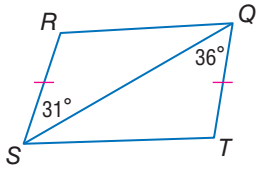


33. **BOATING** Rose and Connor each row across a pond heading to the same point. Neither of them has rowed a boat before, so they both go off course as shown in the diagram. After two minutes, they have each traveled 50 yards. Who is closer to their destination?



Example 6

Compare the given measures.

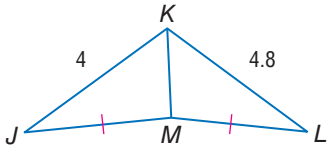


a. RQ and ST

In $\triangle QRS$ and $\triangle STQ$, $\overline{RS} \cong \overline{TQ}$, $\overline{QS} \cong \overline{QS}$, and $\angle SQT > \angle RSQ$. By the Hinge Theorem, $m\angle SQT < m\angle RSQ$, so $RQ < ST$.

b. $m\angle JKM$ and $m\angle LKM$

In $\triangle JKM$ and $\triangle LKM$, $\overline{JM} \cong \overline{LM}$, $\overline{KM} \cong \overline{KM}$, and $LK > JK$. By the Converse of the Hinge Theorem, $\angle LKM > \angle JKM$.

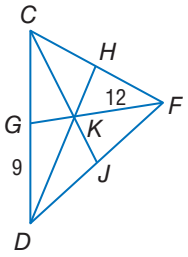


CHAPTER 5

Practice Test

1. GARDENS Maggie wants to plant a circular flower bed within a triangular area set off by three pathways. Which point of concurrency related to triangles would she use for the center of the largest circle that would fit inside the triangle?

In $\triangle CDF$, K is the centroid and $DK = 16$. Find each length.



2. KH

3. CD

4. FG

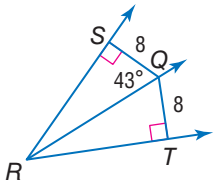
5. PROOF Write an indirect proof.

Given: $5x + 7 \geq 52$

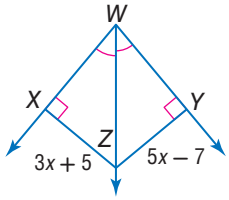
Prove: $x \geq 9$

Find each measure.

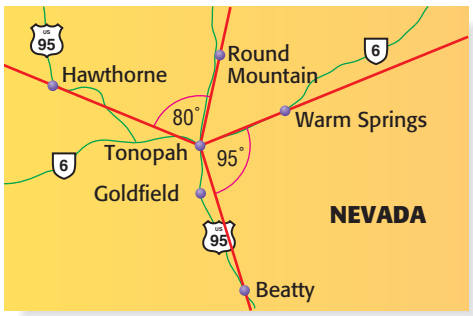
6. $m\angle TQR$



7. XZ



8. GEOGRAPHY The distance from Tonopah to Round Mountain is equal to the distance from Tonopah to Warm Springs. The distance from Tonopah to Hawthorne is the same as the distance from Tonopah to Beatty. Determine which distance is greater, Round Mountain to Hawthorne or Warm Springs to Beatty.

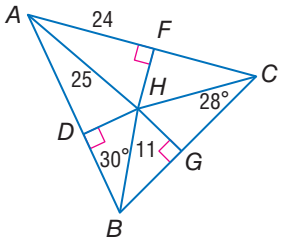


9. MULTIPLE CHOICE If the measures of two sides of a triangle are 3.1 feet and 4.6 feet, which is the *least* possible whole number measure for the third side?

A 1.6 feet **C** 7.5 feet

B 2 feet **D** 8 feet

Point H is the incenter of $\triangle ABC$. Find each measure.



10. DH **11.** BD

12. $m\angle HAC$ **13.** $m\angle DHG$

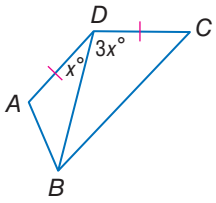
14. MULTIPLE CHOICE If the lengths of two sides of a triangle are 5 and 11, what is the range of possible lengths for the third side?

F $6 < x < 10$ **H** $6 < x < 16$

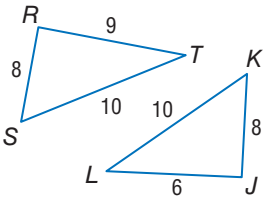
G $5 < x < 11$ **J** $x < 5$ or $x > 11$

Compare the given measures.

15. AB and BC



16. $\angle RST$ and $\angle JKL$



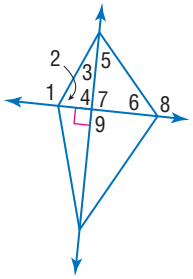
State the assumption necessary to start an indirect proof of each statement.

17. If 8 is a factor of n, then 4 is a factor of n.

18. $m\angle M > m\angle N$

19. If $3a + 7 \leq 28$, then $a \leq 7$.

Use the figure to determine which angle has the greatest measure.



20. $\angle 1$, $\angle 5$, $\angle 6$

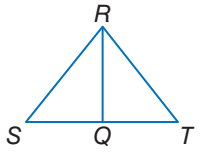
21. $\angle 9$, $\angle 8$, $\angle 3$

22. $\angle 4$, $\angle 3$, $\angle 2$

23. PROOF Write a two-column proof.

Given: $\overline{RQ}$ bisects $\angle SRT$.

Prove: $m\angle SQR > m\angle SRQ$



Find the range for the measure of the third side of a triangle given the measures of the two sides.

24. 10 ft, 16 ft

25. 23 m, 39 m

CHAPTER 5

Preparing for Standardized Tests

Eliminate Unreasonable Answers

You can eliminate unreasonable answers to determine the correct answer when solving multiple choice test items.

Strategies for Eliminating Unreasonable Answers

Step 1

Read the problem statement carefully to determine exactly what you are being asked to find.

- What am I being asked to solve?
- Is the correct answer a whole number, fraction, or decimal?
- Do I need to use a graph or table?
- What units (if any) will the correct answer have?

Step 2

Carefully look over each possible answer choice and evaluate for reasonableness. Do not write any digits or symbols outside the answer boxes.

- Identify any answer choices that are clearly incorrect and eliminate them.
- Eliminate any answer choices that are not in the proper format.
- Eliminate any answer choices that do not have the correct units.

Step 3

Solve the problem and choose the correct answer from those remaining. Check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

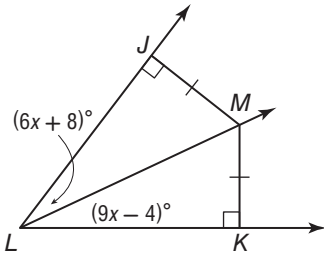
What is the measure of $\angle KLM$?

A 32

B 44

C 78

D 94



Jack Hollingsworth/Photodisc/Getty Images

Read the problem and study the figure carefully. Triangle KLM is a right triangle. Since the sum of the interior angles of a triangle is 180°, $m\angle KLM + m\angle LMK$ must be equal to 90°. Otherwise, the sum would exceed 180°. Since answer choice D is an obtuse angle, it can be eliminated as unreasonable. The correct answer must be A, B, or C.

Solve the problem. According to the converse of the Angle Bisector Theorem, if a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle. Point M is equidistant from rays LJ and LK, so it lies on the angle bisector of $\angle JLK$. Therefore, $\angle JLM$ must be congruent to $\angle KLM$. Set up and solve an equation for x.

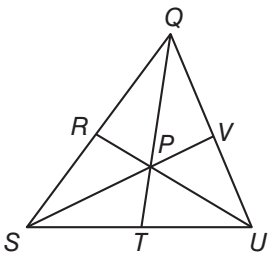
$$6x + 8 = 9x - 4$$
$$-3x = -12$$
$$x = 4$$

So, the measure of $\angle KLM$ is $[9(4) - 4]°$, or 32°. The correct answer is A.

Exercises

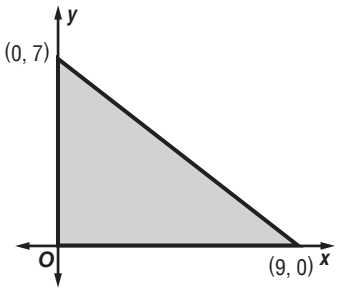
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Point P is the centroid of triangle QUS. If $QP = 14$ centimeters, what is the length of $\overline{QT}$?



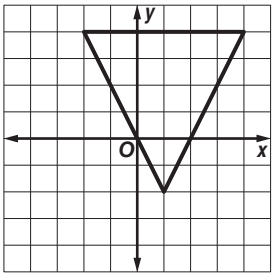
A 7 cm
C 18 cm
B 12 cm
D 21 cm

2. What is the area, in square units, of the triangle shown below?



F 8
H 31.5
G 27.4
J 63

3. What are the coordinates of the orthocenter of the triangle below?



A $\left(-\frac{3}{4}, -1\right)$
C $\left(1, \frac{5}{2}\right)$
B $\left(-\frac{4}{3}, 1\right)$
D $\left(1, \frac{9}{4}\right)$

4. If $\triangle ABC$ is isosceles and $m\angle A = 94$, which of the following *must* be true?

F $m\angle B = 94$
G $m\angle B = 47$
H $AB = BC$
J $AB = AC$

5. Which of the following could *not* be the dimensions of a triangle?

A 1.9, 3.2, 4
C 3, 7.2, 7.5
B 1.6, 3, 4.6
D 2.6, 4.5, 6

CHAPTER 5

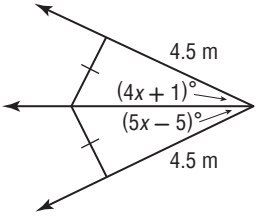
Standardized Test Practice

Cumulative, Chapters 1 through 5

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

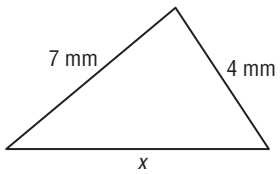
1. Solve for x.



A 3 **C** 5

B 4 **D** 6

2. Which of the following could not be the value of x?



F 8 mm **H** 10 mm

G 9 mm **J** 11 mm

3. Jesse claims that if you live in Lexington, then you live in Kentucky. Which assumption would you need to make to form an indirect proof of this claim?

A Suppose someone lives in Kentucky, but not in Lexington.

B Suppose someone lives in Kentucky and in Lexington.

C Suppose someone lives in Lexington and in Kentucky.

D Suppose someone lives in Lexington, but not in Kentucky.

4. Which of the following best describes the shortest distance from a vertex of a triangle to the opposite side?

F altitude **H** median

G diameter **J** segment

5. Lin started mowing lawns. Let x represent the number of weeks after he began mowing lawns, and y represent the number of customers. Use the points (3, 4) and (9, 6) to find the equation of a line that can be used to predict how many customers Lin has by the end of a certain week.

A $y = \frac{1}{3}x$ **C** $y = \frac{2}{3}x + 2$

B $y = \frac{1}{3}x + 3$ **D** $y = \frac{2}{3}x$

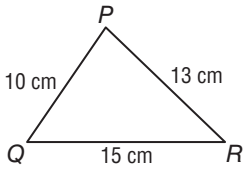
6. What is the correct relationship between the angle measures of $\triangle PQR$?

F $m\angle R < m\angle Q < m\angle P$

G $m\angle R < m\angle P < m\angle Q$

H $m\angle Q < m\angle P < m\angle R$

J $m\angle P < m\angle Q < m\angle R$



7. Which assumption would you need to make in order to start an indirect proof of the statement?

Angle S is not an obtuse angle.

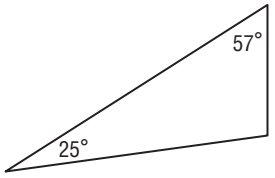
A $\angle S$ is a right angle.

B $\angle S$ is an obtuse angle.

C $\angle S$ is an acute angle.

D $\angle S$ is not an acute angle.

8. Classify the triangle below according to its angle measures.



F acute **H** obtuse

G equiangular **J** right

Test-Taking Tip

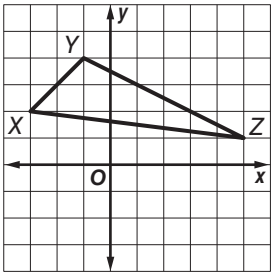
Question 2 The sum of any two sides of a triangle must be greater than the third side.

Short Response/Gridded Response

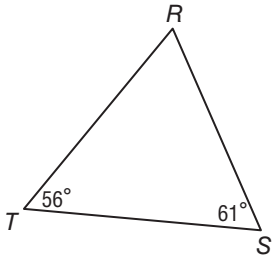
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. GRIDDED RESPONSE If the measures of two sides of a triangle are 9 centimeters and 15 centimeters, what is the least possible measure of the third side in centimeters if the measure is an integer?

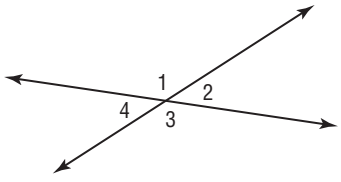
10. What are the coordinates of the orthocenter of the triangle below?



11. List the sides of the triangle below in order from shortest to longest.



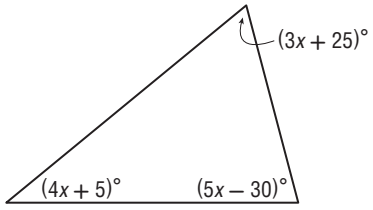
12. Suppose two lines intersect in a plane to form four angles.



What do you know about the pairs of adjacent angles formed? Explain.

13. Eric and Heather are each taking a group of campers hiking in the woods. Eric's group leaves camp and goes 2 miles east, then turns 20° south of east and goes 4 more miles. Heather's group leaves camp and travels 2 miles west, then turns 30° north of west and goes 4 more miles. How many degrees south of east would Eric have needed to turn in order for his group and Heather's group to be the same distance from camp after the two legs of the hike?

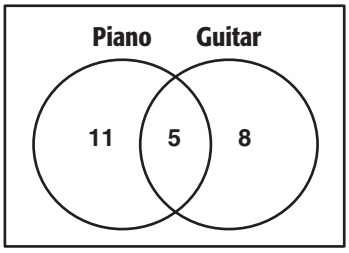
14. GRIDDED RESPONSE Solve for x in the triangle below.



Extended Response

Record your answers on a sheet of paper. Show your work.

15. Refer to the figure to answer each question.



a. How many students play the guitar?

b. How many students play the piano?

c. How many students play both piano and guitar?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	5-1	5-5	5-4	5-2	3-4	5-3	5-4	4-1	5-5	5-2	5-3	1-5	5-6	4-2	2-2