LESSON 12-2

Surface Areas of Prisms and Cylinders

Then
- You found areas of polygons.

Now
1. Find lateral areas and surface areas of prisms.
2. Find lateral areas and surface areas of cylinders.

Why?
- Atlanta's Georgia Aquarium is the largest aquarium in the world, with more than 8 million gallons of water and more than 500 species from around the world. The aquarium has an underwater tunnel that is 100 feet long with 4574 square feet of viewing windows.

NewVocabulary
- lateral face
- lateral edge
- base edge
- altitude
- height
- lateral area
- axis
- composite solid

Common Core State Standards

Content Standards

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

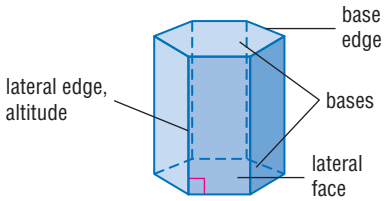
1 Make sense of problems and persevere in solving them.

6 Attend to precision.

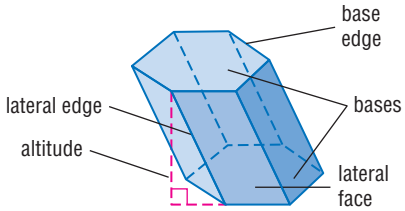
1 Lateral Areas and Surface Areas of Prisms

In a solid figure, faces that are not bases are called **lateral faces**. Lateral faces intersect each other at the **lateral edges**, which are all parallel and congruent. The lateral faces intersect the base at the **base edges**. The **altitude** is a perpendicular segment that joins the planes of the bases. The **height** is the length of the altitude.

Recall that a prism is a polyhedron with two parallel congruent bases. In a right prism, the lateral edges are altitudes and the lateral faces are rectangles. In an oblique prism, the lateral edges are not perpendicular to the bases. At least one lateral face is not a rectangle.

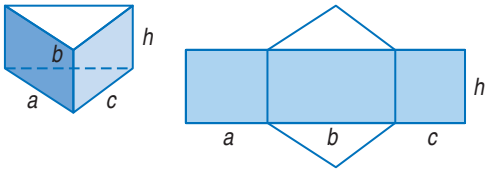


Right Prism



Oblique Prism

The **lateral area** L of a prism is the sum of the areas of the lateral faces. The net at the right shows how to find the lateral area of a prism.



$L = a(h) + b(h) + c(h)$ Sum of areas of lateral faces

$= (a + b + c)h$ Distributive Property

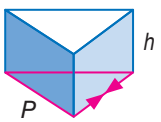
$= Ph$ $P = a + b + c$

KeyConcept Lateral Area of a Prism

Words The lateral area L of a right prism is $L = Ph$, where h is the height of the prism and P is the perimeter of a base.

Symbols $L = Ph$

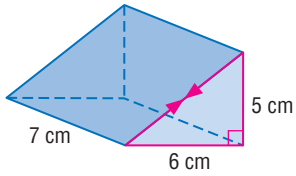
Model



From this point on, you can assume that solids in the text are right solids. If a solid is oblique, it will be clearly stated.

Example 1 Lateral Area of a Prism

Find the lateral area of the prism. Round to the nearest tenth.



Step 1 Find the missing side length of the base.

$c^2 = 6^2 + 5^2$ Pythagorean Theorem

$c^2 = 61$ Simplify.

$c \approx 7.8$ Take the positive square root of each side.

Step 2 Find the lateral area.

$L = Ph$ Lateral area of a prism

$\approx (5 + 6 + 7.8)7$ Substitution

≈ 131.6 Simplify.

The lateral area is about 131.6 square centimeters.

WatchOut!
Right Prisms The bases of a right prism are congruent, but the faces are not always congruent.

GuidedPractice

1. The length of each side of the base of a regular octagonal prism is 6 inches, and the height is 11 inches. Find the lateral area.

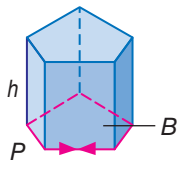
The surface area of a prism is the sum of the lateral area and the areas of the bases.

KeyConcept Surface Area of a Prism

Words The surface area S of a right prism is $S = L + 2B$, where L is its lateral area and B is the area of a base.

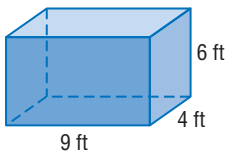
Symbols $S = L + 2B$ or $S = Ph + 2B$

Model



Example 2 Surface Area of a Prism

Find the surface area of the rectangular prism.



Use the 9-foot by 4-foot rectangle as the base.

$S = Ph + 2B$ Surface area of a prism

$= (2 \cdot 9 + 2 \cdot 4)(6) + 2(9 \cdot 4)$ Substitution

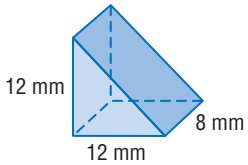
$= 228$ Simplify.

The surface area of the prism is 228 square feet.

StudyTip
CCSS Perseverance In Example 2, you can also use a 6-foot by 4-foot rectangle as the base. The height would be 9 feet. While choosing a different base does not affect the surface area, it will change the lateral area.

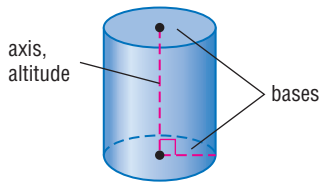
GuidedPractice

2. Find the surface area of the triangular prism. Round to the nearest tenth.

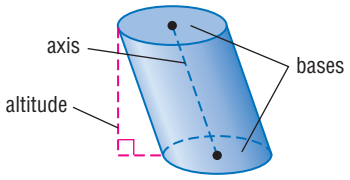


2 Lateral Areas and Surface Areas of Cylinders

The **axis** of a cylinder is the segment with endpoints that are centers of the circular bases. If the axis is also an altitude, then the cylinder is a right cylinder. If the axis is not an altitude, then the cylinder is an oblique cylinder.

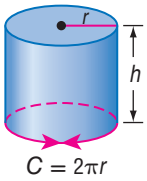


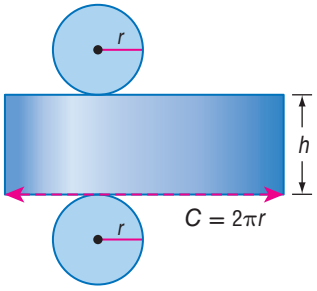
Right Cylinder



Oblique Cylinder

The lateral area of a right cylinder is the area of the curved surface. Like a right prism, the lateral area L equals Ph. Since the base is a circle, the perimeter is the circumference of the circle C. So, the lateral area is Ch or $2\pi rh$.





StudyTip

Formulas An alternate formula for the lateral area of a cylinder is $L = \pi dh$, with πd as the circumference of a circle.

The surface area of a cylinder is the lateral area plus the areas of the bases.

KeyConcept Surface Area of a Cylinder

Words

The lateral area L of a right cylinder is $L = 2\pi rh$, where r is the radius of a base and h is the height.

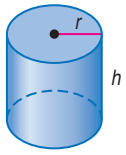
The surface area S of a right cylinder is $S = 2\pi rh + 2\pi r^2$, where r is the radius of a base and h is the height.

Symbols

$L = 2\pi rh$

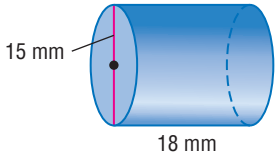
$S = L + 2B$ or $2\pi rh + 2\pi r^2$

Model



Example 3 Lateral Area and Surface Area of a Cylinder

Find the lateral area and the surface area of the cylinder. Round to the nearest tenth.



$L = 2\pi rh$	Lateral area of a cylinder
$= 2\pi(7.5)(18)$	Replace r with 7.5 and h with 18.
≈ 848.2	Use a calculator.
$S = 2\pi rh + 2\pi r^2$	Surface area of a cylinder
$\approx 848.2 + 2\pi(7.5)^2$	Replace $2\pi rh$ with 848.2 and r with 7.5.
≈ 1201.6	Use a calculator.

The lateral area is about 848.2 square millimeters, and the surface area is about 1201.6 square millimeters.

StudyTip

Estimation Before finding the lateral area of a cylinder, use mental math to estimate. To estimate, multiply the diameter by 3 (to approximate π) and then by the height of the cylinder.

GuidedPractice

3A. $r = 5$ in., $h = 9$ in.

3B. $d = 6$ cm, $h = 4.8$ cm

Real-World Example 4 Find Missing Dimensions

CRAFTS **Sheree used the rectangular piece of felt shown at the right to cover the curved surface of her cylindrical pencil holder. What is the radius of the pencil holder?**

$L = 2\pi rh$	Lateral area of a cylinder
$63 = 2\pi r(5)$	Replace L with 12.6 • 5 or 63 and h with 5.
$63 = 10\pi r$	Simplify.
$2.0 \approx r$	Divide each side by 10π.

The radius of the pencil holder is about 2 inches.

GuidedPractice

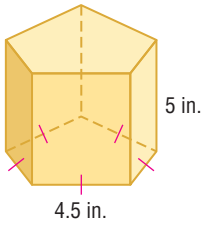
4. Find the diameter of a base of a cylinder if the surface area is 464π square centimeters and the height is 21 centimeters.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

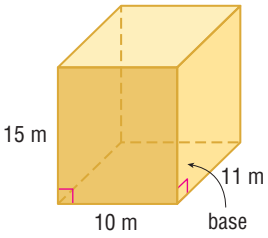
Example 1

1. Find the lateral area of the prism.



Examples 1–2 **Find the lateral area and surface area of each prism.**

2.



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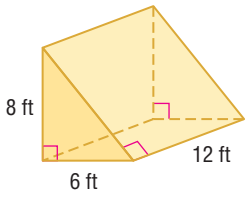
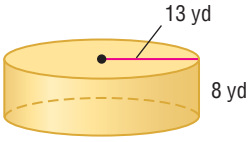


Example 3

4. CARS Evan is buying new tire rims that are 14 inches in diameter and 6 inches wide. Determine the lateral area of each rim. Round to the nearest tenth.

Find the lateral area and surface area of each cylinder. Round to the nearest tenth.

5.



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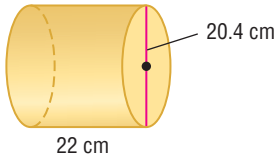
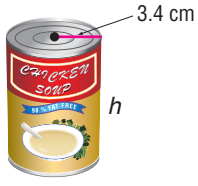


Example 4

7. FOOD The can of soup at the right has a surface area of 286.3 square centimeters. What is the height of the can? Round to the nearest tenth.



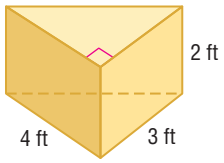
8. The surface area of a cube is 294 square inches. Find the length of a lateral edge.

Practice and Problem Solving

Extra Practice is on page R12.

Examples 1–2 **Find the lateral area and surface area of each prism. Round to the nearest tenth if necessary.**

9



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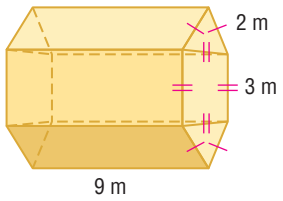

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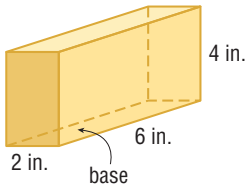

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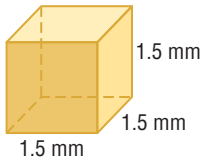

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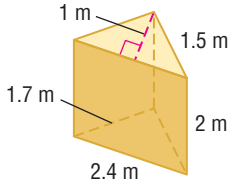

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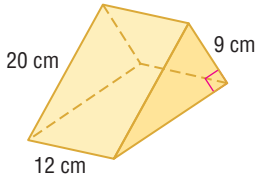


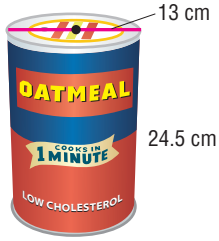
15. rectangular prism: $\ell = 25$ centimeters, $w = 18$ centimeters, $h = 12$ centimeters

16. triangular prism: $h = 6$ inches, right triangle base with legs 9 inches and 12 inches

Examples 1–3 **CEREAL** **Find the lateral area and the surface area of each cereal container. Round to the nearest tenth if necessary.**

17.

18.



Example 3 **CCSS SENSE-MAKING** **Find the lateral area and surface area of each cylinder. Round to the nearest tenth.**

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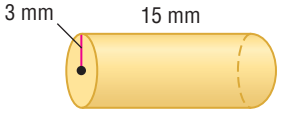

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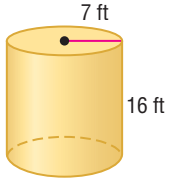

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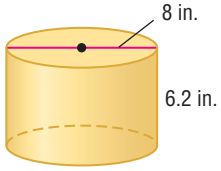

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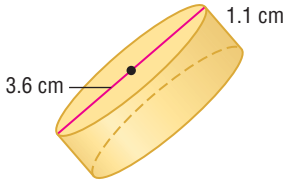
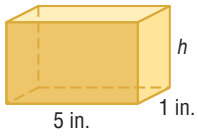


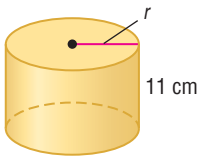
23. **WORLD RECORDS** The largest beverage can was a cylinder with height 4.67 meters and diameter 2.32 meters. What was the surface area of the can to the nearest tenth?

Example 4 **Use the given lateral area and the diagram to find the missing measure of each solid. Round to the nearest tenth if necessary.**

24. $L = 48 \text{ in}^2$



25. $L \approx 635.9 \text{ cm}^2$



26. A right rectangular prism has a surface area of 1020 square inches, a length of 6 inches, and a width of 9 inches. Find the height.

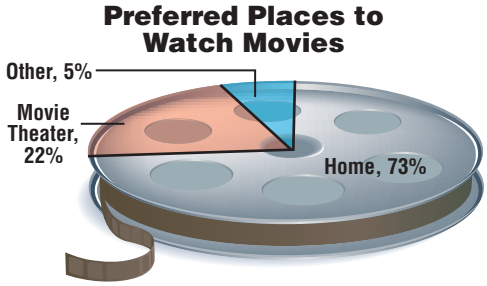
27 A cylinder has a surface area of 256π square millimeters and a height of 8 millimeters. Find the diameter.

28. MONUMENTS A *monolith* mysteriously appeared overnight at Seattle, Washington's Manguson Park. A hollow rectangular prism, the monolith was 9 feet tall, 4 feet wide, and 1 foot deep.

a. Find the area in square feet of the structure's surfaces that lie above the ground.

b. Use dimensional analysis to find the area in square yards.

29. ENTERTAINMENT The graphic shows the results of a survey in which people were asked where they like to watch movies.

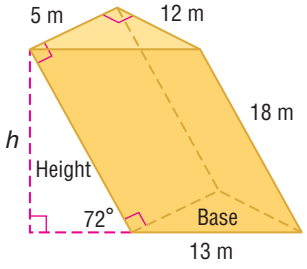


a. Suppose the film can is a cylinder 12 inches in diameter. Explain how to find the surface area of the portion that represents people who prefer to watch movies at home.

b. If the film can is 3 inches tall, find the surface area of the portion in part **a**.

CCSS **SENSE-MAKING** **Find the lateral area and surface area of each oblique solid. Round to the nearest tenth.**

30.



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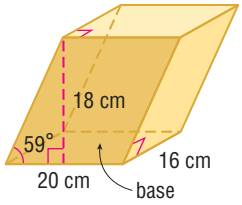


32. LAMPS The lamp shade is a cylinder of height 18 inches with a diameter of $6\frac{3}{4}$ inches.

a. What is the lateral area of the shade to the nearest tenth?

b. How does the lateral area change if the height is divided by 2?

33. Find the approximate surface area of a right hexagonal prism if the height is 9 centimeters and each base edge is 4 centimeters. (*Hint*: First, find the length of the apothem of the base.)

Cheryl Hatch/AP Images

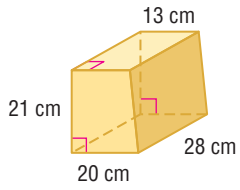
34. DESIGN A mailer needs to hold a poster that is almost 38 inches long and has a maximum rolled diameter of 6 inches.

a. Design a mailer that is a triangular prism. Sketch the mailer and its net.

b. Suppose you want to minimize the surface area of the mailer. What would be the dimensions of the mailer and its surface area?

A composite solid is a three-dimensional figure that is composed of simpler figures. Find the surface area of each composite solid. Round to the nearest tenth if necessary.

35



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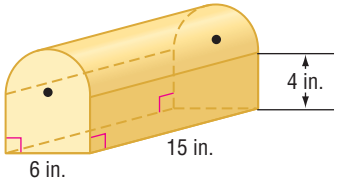

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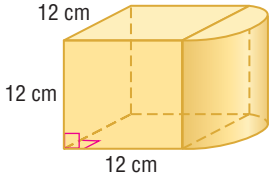


38. MULTIPLE REPRESENTATIONS In this problem, you will investigate the lateral area and surface area of a cylinder.

a. Geometric Sketch cylinder *A* with a radius of 3 centimeters and a height of 5 centimeters, cylinder *B* with a radius of 6 centimeters and a height of 5 centimeters, and cylinder *C* with a radius of 3 centimeters and a height of 10 centimeters.

b. Tabular Create a table of the radius, height, lateral area, and surface area of cylinders A, B, and C. Write the areas in terms of π.

c. Verbal If the radius is doubled, what effect does it have on the lateral area and the surface area of a cylinder? If the height is doubled, what effect does it have on the lateral area and the surface area of a cylinder?

H.O.T. Problems Use Higher-Order Thinking Skills

39. ERROR ANALYSIS Montell and Derek are finding the surface area of a cylinder with height 5 centimeters and radius 6 centimeters. Is either of them correct? Explain.

Montell

$S = \pi(6)^2 + \pi(6)(5)$

$= 36\pi + 30\pi$

$= 66\pi \text{ cm}^2$

Derek

$S = 2\pi(6)^2 + 2\pi(6)(5)$

$= 72\pi + 60\pi$

$= 132\pi \text{ cm}^2$

40. WRITING IN MATH Sketch an oblique rectangular prism, and describe the shapes that would be included in a net for the prism. Explain how the net is different from that of a right rectangular prism.

41. CCSS PRECISION Compare and contrast finding the surface area of a prism and finding the surface area of a cylinder.

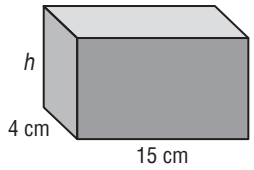
42. OPEN ENDED Give an example of two cylinders that have the same lateral area and different surface areas. Describe the lateral area and surface areas of each.

43. CHALLENGE A right prism has a height of h units and a base that is an equilateral triangle of side ℓ units. Find the general formula for the total surface area of the prism. Explain your reasoning.

44. WRITING IN MATH A square based prism and a triangular prism are the same height. The base of the triangular prism is an equilateral triangle, with an altitude equal in length to the side of the square. Compare the lateral areas of the prisms.

Standardized Test Practice

45. If the surface area of the right rectangular prism is 310 square centimeters, what is the measure of the height h of the prism?



A 5 cm
B $5\frac{1}{6}$ cm
C 10
D $13\frac{3}{9}$ cm

46. SHORT RESPONSE A cylinder has a circumference of 16π inches and a height of 20 inches. What is the surface area of the cylinder in terms of π?

47. Parker Flooring charges the following to install a hardwood floor in a new home.
Subflooring: \$2.25 per square foot
Wood flooring : \$4.59 per square foot
Baseboards: \$1.95 per linear foot around room
Nail & other materials: \$25.95 per job
Labor: \$99 plus \$0.99 square foot
What is the cost to install hardwood flooring in a room that is 18 by 15 feet?

F \$2169.75
G \$2268.75
H \$2367.75
J \$2765.55

48. SAT/ACT What is the value of $f(-2)$ if $f(x) = x^3 + 4x^2 - 2x - 3$?

A -31
B $-\frac{9}{2}$
C 9
D 25
E 28

Spiral Review

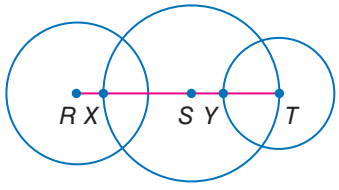
Use isometric dot paper to sketch each prism. (Lesson 12-1)

49. rectangular prism 2 units high, 3 units long, and 2 units wide

50. triangular prism 2 units high with bases that are right triangles with legs 3 units and 4 units long

51. BAKING A bakery sells single-layer mini-cakes that are 3 inches in diameter for \$4 each. They also have a cake with the same thickness and a 9-inch diameter for \$15. Compare the areas of the cake tops to determine, which option is a better buy, nine mini-cakes or one 9-inch cake. Explain. (Lesson 11-5)

The diameters of $\odot R$, $\odot S$, and $\odot T$ are 10 inches, 14 inches, and 9 inches, respectively. Find each measure. (Lesson 10-1)



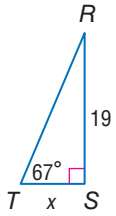
52. YX

53. SY

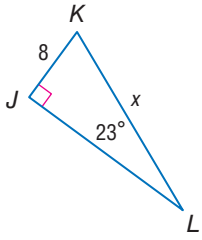
Skills Review

Find x. Round to the nearest tenth.

54.



55.



56.