LESSON 12-4

Volumes of Prisms and Cylinders

Then
- You found surface areas of prisms and cylinders.

Now
1. Find volumes of prisms.
2. Find volumes of cylinders.

Why?
- Planters come in a variety of shapes and sizes. You can approximate the amount of soil needed to fill a planter by finding the volume of the three-dimensional figure that it most resembles.

Common Core State Standards

Content Standards

G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

Mathematical Practices

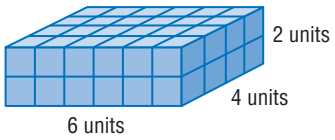
1 Make sense of problems and persevere in solving them.

7 Look for and make use of structure.

1 Volume of Prisms

Recall that the volume of a solid is the measure of the amount of space the solid encloses. Volume is measured in cubic units.

The rectangular prism at the right has 6 • 4 or 24 cubic units in the bottom layer. Since there are two layers, the total volume is 24 • 2 or 48 cubic units.

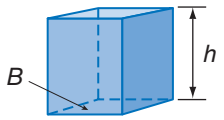


KeyConcept Volume of a Prism

Words The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height of the prism.

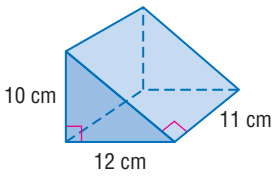
Symbols $V = Bh$

Model



Example 1 Volume of a Prism

Find the volume of the prism.



Step 1 Find the area of the base B.

$B = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(12)(10)$ or 60 $b = 12$ and $h = 10$

Step 2 Find the volume of the prism.

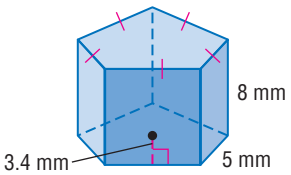
$V = Bh$ Volume of a prism

$= 60(11)$ or 660 $B = 60$ and $h = 11$

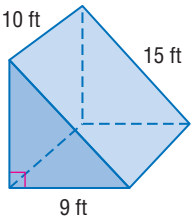
The volume of the prism is 660 cubic centimeters.

GuidedPractice

1A.



1B.



Real-World Career

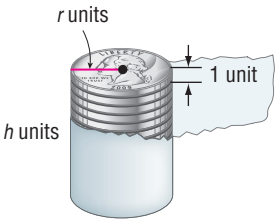
Architectural Engineer

An architectural engineer applies the technical skills of engineering to the design, construction, operation, maintenance, and renovation of buildings.

Architectural engineers are required to have a bachelor's degree in engineering along with specialized coursework. Refer to Exercise 35.

2 Volume of Cylinders

Like a prism, the volume of a cylinder can be thought of as consisting of layers. For a cylinder, these layers are congruent circular discs, similar to the coins in the roll shown. If we interpret the area of the base as the volume of a one-unit-high layer and the height of the cylinder as the number of layers, then the volume of the cylinder is equal to the volume of a layer times the number of layers or the area of the base times the height.

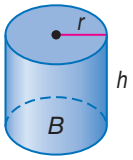


KeyConcept Volume of a Cylinder

Words The volume V of a cylinder is $V = Bh$ or $V = \pi r^2 h$, where B is the area of the base, h is the height of the cylinder, and r is the radius of the base.

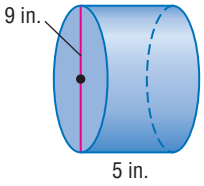
Symbols $V = Bh$ or $V = \pi r^2 h$

Model



Example 2 Volume of a Cylinder

Find the volume of the cylinder at the right.



Estimate: $V \approx 3 \cdot 5^2 \cdot 5$ or 375 in^3

$V = \pi r^2 h$ Volume of a cylinder

$= \pi(4.5)^2(5)$ $r = 4.5$ and $h = 5$

≈ 318.1 Use a calculator.

The volume of the cylinder is about 318.1 cubic inches. This is fairly close to the estimate, so the answer is reasonable.

GuidedPractice

2. Find the volume of a cylinder with a radius of 3 centimeters and a height of 8 centimeters. Round to the nearest tenth.

The first group of books at the right represents a right prism. The second group represents an oblique prism. Both groups have the same number of books. If all the books are the same size, then the volume of both groups is the same.

This demonstrates the following principle, which applies to all solids.

WatchOut!

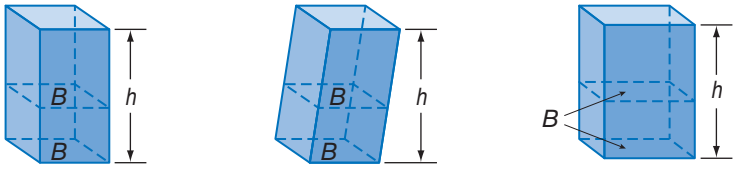
Cross-Sectional Area

For solids with the same height to have the same volume, their cross-sections must have the same area. The cross sections of the different solids do not have to be congruent polygons.

KeyConcept Cavalieri's Principle

Words If two solids have the same height h and the same cross-sectional area B at every level, then they have the same volume.

Models



These prisms all have a volume of Bh.

Javier Larrea/age fotostock

Problem-Solving Tip

Make a Model When solving problems involving volume of solids, one way to help you visualize the problem is to make a model of the solid.

Example 3 Volume of an Oblique Solid

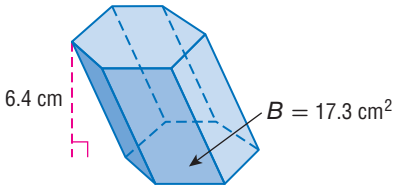
Find the volume of an oblique hexagonal prism if the height is 6.4 centimeters and the base area is 17.3 square centimeters.

$V = Bh$ Volume of a prism

$= 17.3(6.4)$ $B = 17.3$ and $h = 6.4$

$= 110.72$ Simplify.

The volume is 110.72 cubic centimeters.

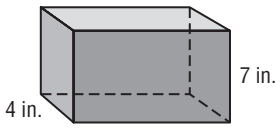


GuidedPractice

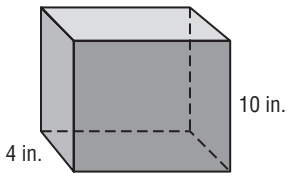
3. Find the volume of an oblique cylinder that has a radius of 5 feet and a height of 3 feet. Round to the nearest tenth.

Standardized Test Example 4 Comparing Volumes of Solids

Prisms A and B have the same length and width, but different heights. If the volume of Prism B is 150 cubic inches greater than the volume of Prism A, what is the length of each prism?



Prism A



Prism B

A 10 in. **B** $11\frac{1}{2}$ in. **C** 12 in. **D** $12\frac{1}{2}$ in.

Read the Test Item

You know two dimensions of each solid and that the difference between their volumes is 150 cubic inches.

Test-Taking Tip

Writing Equations By choosing a variable to represent the unknown and setting up an equation, you make it easier to check for mistakes as you work out a problem.

Solve the Test Item

Volume of Prism B − Volume of Prism A = 150 Write an equation.

$4\ell \cdot 10 - 4\ell \cdot 7 = 150$ Use $V = Bh$.

$12\ell = 150$ Simplify.

$\ell = 12\frac{1}{2}$ Divide each side by 12.

The length of each prism is $12\frac{1}{2}$ inches. The correct answer is D.

GuidedPractice

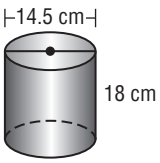
4. The containers at the right are filled with popcorn. About how many times as much popcorn does the larger container hold?

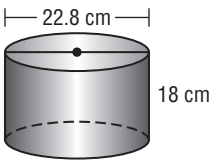
F 1.6 times as much

G 2.5 times as much

H 3.3 times as much

J 5.0 times as much



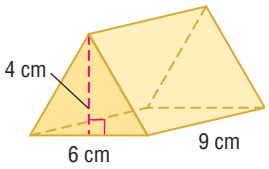


Check Your Understanding

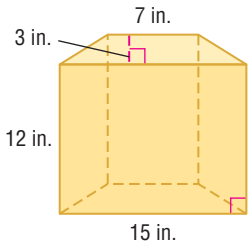
= Step-by-Step Solutions begin on page R14.

Examples 1 and 3 Find the volume of each prism.

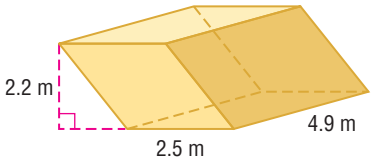
1.



2.



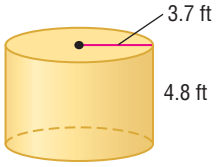
3. the oblique rectangular prism shown at the right



4. an oblique pentagonal prism with a base area of 42 square centimeters and a height of 5.2 centimeters

Examples 2–3 Find the volume of each cylinder. Round to the nearest tenth.

5.



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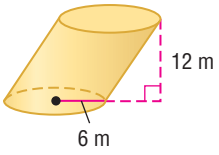


7. a cylinder with a diameter of 16 centimeters and a height of 5.1 centimeters

8. a cylinder with a radius of 4.2 inches and a height of 7.4 inches

Example 4

9. MULTIPLE CHOICE A rectangular lap pool measures 80 feet long by 20 feet wide. If it needs to be filled to four feet deep and each cubic foot holds 7.5 gallons, how many gallons will it take to fill the lap pool?

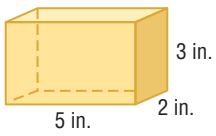
A 4000 **B** 6400 **C** 30,000 **D** 48,000

Practice and Problem Solving

Extra Practice is on page R12.

Examples 1 and 3 CCSS **SENSE-MAKING** Find the volume of each prism.

10.



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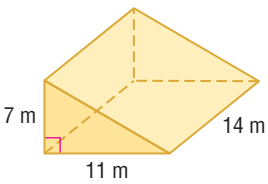

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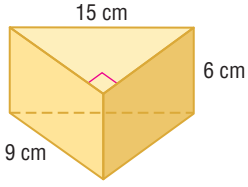

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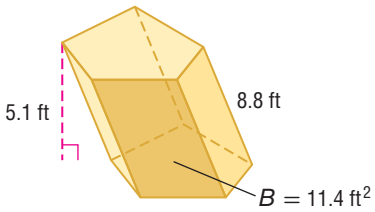
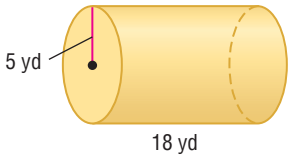


14. an oblique hexagonal prism with a height of 15 centimeters and with a base area of 136 square centimeters

15. a square prism with a base edge of 9.5 inches and a height of 17 inches

Examples 2–3 CCSS **SENSE-MAKING** **Find the volume of each cylinder. Round to the nearest tenth.**

16.



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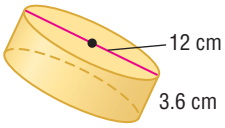
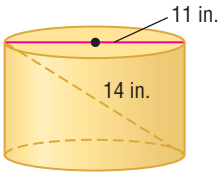


18.



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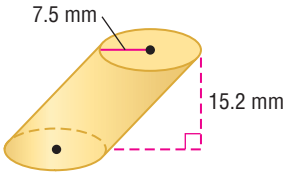
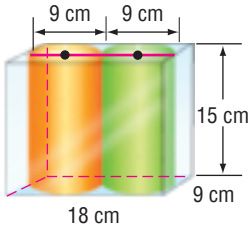


Example 4

20. PLANTER A planter is in the shape of a rectangular prism 18 inches long, $14\frac{1}{2}$ inches deep, and 12 inches high. What is the volume of potting soil in the planter if the planter is filled to $1\frac{1}{2}$ inches below the top?

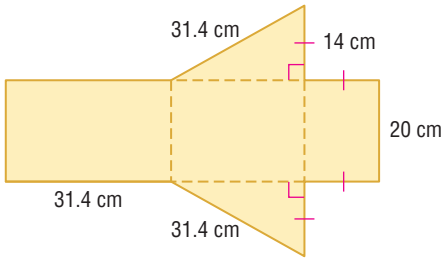
21. SHIPPING A box 18 centimeters by 9 centimeters by 15 centimeters is being used to ship two cylindrical candles. Each candle has a diameter of 9 centimeters and a height of 15 centimeters, as shown at the right. What is the volume of the empty space in the box?



22. SANDCASTLES In a sandcastle competition, contestants are allowed to use only water, shovels, and 10 cubic feet of sand. To transport the correct amount of sand, they want to create cylinders that are 2 feet tall to hold enough sand for one contestant. What should the diameter of the cylinders be?

Find the volume of the solid formed by each net.

23.



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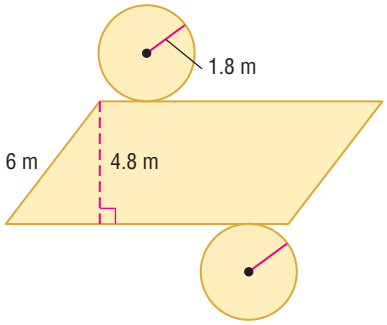
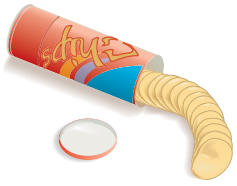


25. FOOD A cylindrical can of baked potato chips has a height of 27 centimeters and a radius of 4 centimeters. A new can is advertised as being 30% larger than the regular can. If both cans have the same radius, what is the height of the larger can?



26. CHANGING DIMENSIONS A cylinder has a radius of 5 centimeters and a height of 8 centimeters. Describe how each change affects the volume of the cylinder.

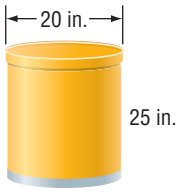
a. The height is tripled.

b. The radius is tripled.

c. Both the radius and the height are tripled.

d. The dimensions are exchanged.

27. SOIL A soil scientist wants to determine the bulk density of a potting soil to assess how well a specific plant will grow in it. The density of the soil sample is the ratio of its weight to its volume.



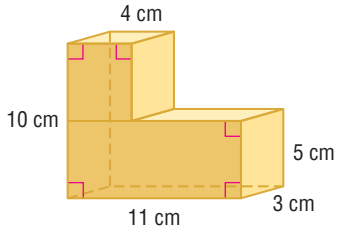
a. If the weight of the container with the soil is 20 pounds and the weight of the container alone is 5 pounds, what is the soil's bulk density?

b. Assuming that all other factors are favorable, how well should a plant grow in this soil if a bulk density of 0.0018 pound per square inch is desirable for root growth? Explain.

c. If a bag of this soil holds 2.5 cubic feet, what is its weight in pounds?

Find the volume of each composite solid. Round to the nearest tenth if necessary.

28.



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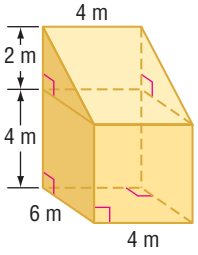

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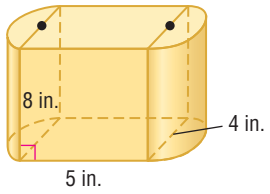
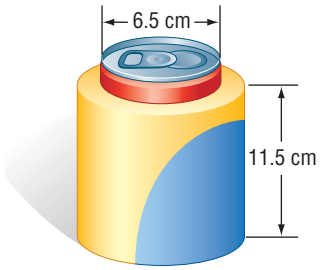


31. MANUFACTURING A can 12 centimeters tall fits into a rubberized cylindrical holder that is 11.5 centimeters tall, including 1 centimeter for the thickness of the base of the holder. The thickness of the rim of the holder is 1 centimeter. What is the volume of the rubberized material that makes up the holder?



Find each measure to the nearest tenth.

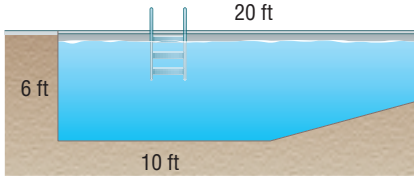
32. A cylindrical can has a volume of 363 cubic centimeters. The diameter of the can is 9 centimeters. What is the height?

33. A cylinder has a surface area of 144π square inches and a height of 6 inches. What is the volume?

34. A rectangular prism has a surface area of 432 square inches, a height of 6 inches, and a width of 12 inches. What is the volume?

35. ARCHITECTURE A cylindrical stainless steel column is used to hide a ventilation system in a new building. According to the specifications, the diameter of the column can be between 30 centimeters and 95 centimeters. The height is to be 500 centimeters. What is the difference in volume between the largest and smallest possible column? Round to the nearest tenth cubic centimeter.

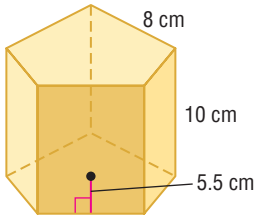
36. CCSS MODELING The base of a rectangular swimming pool is sloped so one end of the pool is 6 feet deep and the other end is 3 feet deep, as shown in the figure. If the width is 15 feet, find the volume of water it takes to fill the pool.



37. CHANGING DIMENSIONS A soy milk company is planning a promotion in which the volume of soy milk in each container will be increased by 25%. The company wants the base of the container to stay the same. What will be the height of the new containers?

38. DESIGN Sketch and label (in inches) three different designs for a dry ingredient measuring cup that holds 1 cup. Be sure to include the dimensions in each drawing. (1 cup ≈ 14.4375 in^3)

39 Find the volume of the regular pentagonal prism at the right by dividing it into five equal triangular prisms. Describe the base area and height of each triangular prism.



40. PATIOS Mr. Thomas is planning to remove an old patio and install a new rectangular concrete patio 20 feet long, 12 feet wide, and 4 inches thick. One contractor bid $2225 for the project. A second contractor bid $500 per cubic yard for the new patio and $700 for removal of the old patio. Which is the less expensive option? Explain.

41. MULTIPLE REPRESENTATIONS In this problem, you will investigate cylinders.

a. Geometric Draw a right cylinder and an oblique cylinder with a height of 10 meters and a diameter of 6 meters.

b. Verbal A square prism has a height of 10 meters and a base edge of 6 meters. Is its volume greater than, less than, or equal to the volume of the cylinder? Explain.

c. Analytical Describe which change affects the volume of the cylinder more: multiplying the height by x or multiplying the radius by x. Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

42. CCSS CRITIQUE Francisco and Valerie each calculated the volume of an equilateral triangular prism with an apothem of 4 units and height of 5 units. Is either of them correct? Explain your reasoning.

Francisco

$V = Bh$
$= \frac{1}{2}aP \cdot h$
$= \frac{1}{2}(4)(24\sqrt{3}) \cdot 5$
$= 240\sqrt{3}$ cubic units

Valerie

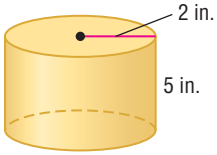
$V = Bh$
$= \frac{\sqrt{3}}{2}s^2 \cdot h$
$= \frac{\sqrt{3}}{2}(4\sqrt{3})^2 \cdot 5$
$= 120\sqrt{3}$ cubic units

43. CHALLENGE The cylindrical can below is used to fill a container with liquid. It takes three full cans to fill the container. Describe possible dimensions of the container if it is each of the following shapes.

a. rectangular prism

b. square prism

c. triangular prism with a right triangle as the base



44. WRITING IN MATH Write a helpful response to the following question posted on an Internet gardening forum.

I am new to gardening. The nursery will deliver a truckload of soil, which they say is 4 yards. I know that a yard is 3 feet, but what is a yard of soil? How do I know what to order?

45. OPEN ENDED Draw and label a prism that has a volume of 50 cubic centimeters.

46. REASONING Determine whether the following statement is true or false. Explain.

Two cylinders with the same height and the same lateral area must have the same volume.

47. WRITING IN MATH How are the volume formulas for prisms and cylinders similar? How are they different?

Standardized Test Practice

48. The volume of a triangular prism is 1380 cubic centimeters. Its base is a right triangle with legs measuring 8 centimeters and 15 centimeters. What is the height of the prism?

A 34.5 cm **C** 17 cm

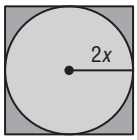
B 23 cm **D** 11.5 cm

49. A cylindrical tank used for oil storage has a height that is half the length of its radius. If the volume of the tank is 1,122,360 ft^3, what is the tank's radius?

F 89.4 ft **H** 280.9 ft

G 178.8 ft **J** 561.8 ft

50. SHORT RESPONSE What is the ratio of the area of the circle to the area of the square?



51. SAT/ACT A county proposes to enact a new 0.5% property tax. What would be the additional tax amount for a landowner whose property has a taxable value of $85,000?

A $4.25 **D** $4250

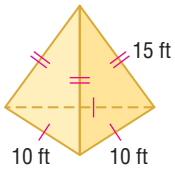
B $170 **E** $42,500

C $425

Spiral Review

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth if necessary. (Lesson 12-3)

52.



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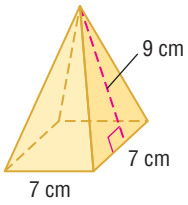

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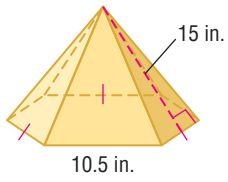
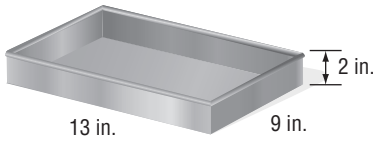


55. BAKING Many baking pans are given a special nonstick coating. A rectangular cake pan is 9 inches by 13 inches by 2 inches deep. What is the area of the inside of the pan that needs to be coated? (Lesson 12-2)



Find the indicated measure. Round to the nearest tenth. (Lesson 11-3)

56. The area of a circle is 54 square meters. Find the diameter.

57. Find the diameter of a circle with an area of 102 square centimeters.

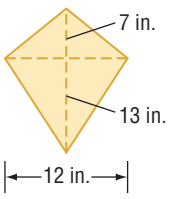
58. The area of a circle is 191 square feet. Find the radius.

59. Find the radius of a circle with an area of 271 square inches.

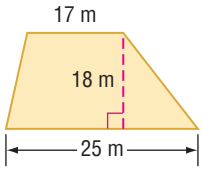
Skills Review

Find the area of each trapezoid, rhombus, or kite.

60.



61.



62.