LESSON 12-3 Surface Areas of Pyramids and Cones

Then

- You found areas of regular polygons.

Now

1. Find lateral areas and surface areas of pyramids.
2. Find lateral areas and surface areas of cones.

Why?

- The Transamerica Pyramid in San Francisco, California, covers nearly one city block. Its unconventional design allows light and air to filter down to the streets around the building, unlike the more traditional rectangular prism skyscrapers.

NewVocabulary
regular pyramid
slant height
right cone
oblique cone

Common Core State Standards

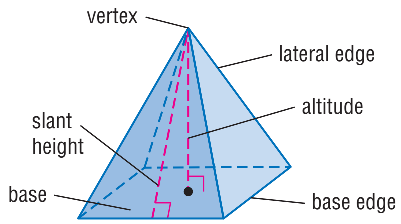
Content Standards
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★

Mathematical Practices
1 Make sense of problems and persevere in solving them.
6 Attend to precision.

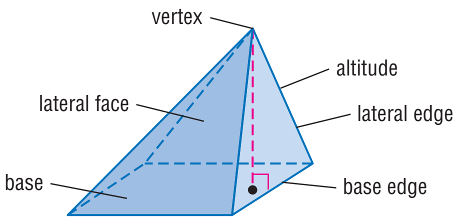
1 Lateral Area and Surface Area of Pyramids

The *lateral faces* of a pyramid intersect at a common point called the *vertex*. Two lateral faces intersect at a *lateral edge*. A lateral face and the base intersect at a *base edge*. The *altitude* is the segment from the vertex perpendicular to the base.

A **regular pyramid** has a base that is a regular polygon and the altitude has an endpoint at the center of the base. All the lateral edges are congruent and all the lateral faces are congruent isosceles triangles. The height of each lateral face is called the **slant height** ℓ of the pyramid.

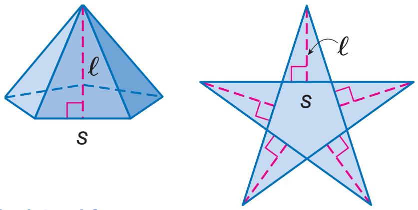


Regular Pyramid



Nonregular Pyramid

The lateral area L of a regular pentagonal pyramid is the sum of the areas of all its congruent triangular faces as shown in the net at the right.



$$L = \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell$$ Sum of the areas of the lateral faces

$$= \frac{1}{2}\ell(s + s + s + s + s)$$ Distributive Property

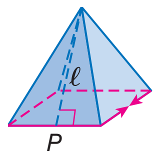
$$= \frac{1}{2}P\ell$$ $P = s + s + s + s + s$

KeyConcept Lateral Area of a Regular Pyramid

Words The lateral area L of a regular pyramid is $L = \frac{1}{2}P\ell$, where ℓ is the slant height and P is the perimeter of the base.

Symbols $L = \frac{1}{2}P\ell$

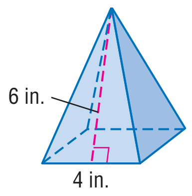
Model



> **StudyTip**
> **Alternative Method** You can also find the lateral area of a pyramid by adding the areas of the congruent lateral faces.
> area of one face:
> $\frac{1}{2}(4)(6) = 12 \text{ in}^2$
> lateral area:
> $4 \cdot 12 = 48 \text{ in}^2$

Example 1 Lateral Area of a Regular Pyramid

Find the lateral area of the square pyramid.



$L = \frac{1}{2}P\ell$ — Lateral area of a regular pyramid

$= \frac{1}{2}(16)(6)$ or 48 — $P = 4 \cdot 4$ or 16, $\ell = 6$

The lateral area is 48 square inches.

GuidedPractice

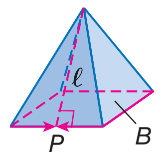
1. Find the lateral area of a regular hexagonal pyramid with a base edge of 9 centimeters and a lateral height of 7 centimeters.

The surface area of a pyramid is the sum of the lateral area and the area of the base.

KeyConcept Surface Area of a Regular Pyramid

Words The surface area S of a regular pyramid is $S = \frac{1}{2}P\ell + B$, where P is the perimeter of the base, ℓ is the slant height, and B is the area of the base.

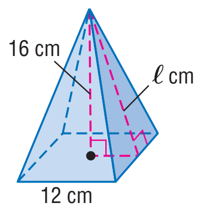
Model



Symbols $S = \frac{1}{2}P\ell + B$

Example 2 Surface Area of a Square Pyramid

Find the surface area of the square pyramid to the nearest tenth.



Step 1 Find the slant height.

$c^2 = a^2 + b^2$ — Pythagorean Theorem

$\ell^2 = 16^2 + 6^2$ — $a = 16$, $b = 6$, and $c = \ell$

$\ell = \sqrt{292}$ — Simplify.

Step 2 Find the perimeter and area of the base.

$P = 4 \cdot 12$ or 48 cm $\quad A = 12^2$ or 144 cm^2

Step 3 Find the surface area of the pyramid.

$S = \frac{1}{2}P\ell + B$ — Surface area of a regular pyramid

$= \frac{1}{2}(48)\sqrt{292} + 144$ — $P = 48$, $\ell = \sqrt{292}$, and $B = 144$

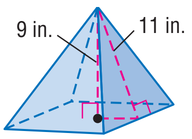
≈ 554.1 — Use a calculator.

The surface area of the pyramid is about 554.1 square centimeters.

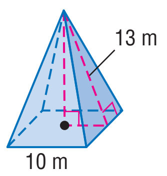
> **StudyTip**
> **Making Connections** The surface area of a pyramid equals $L + B$, not $L + 2B$, because a pyramid has only one base.

GuidedPractice

2A.

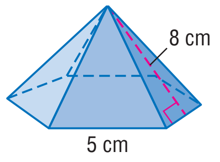


2B.



Example 3 Surface Area of a Regular Pyramid

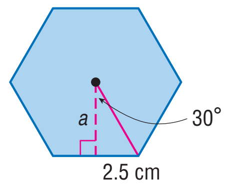
Find the surface area of the regular pyramid. Round to the nearest tenth.



Step 1 Find the perimeter of the base.

$P = 6 \cdot 5$ or 30 cm

Step 2 Find the length of the apothem and the area of the base.



A central angle of the hexagon is $\frac{360°}{6}$ or 60°, so the angle formed in the triangle at the right is 30°.

$\tan 30° = \frac{2.5}{a}$	Write a trigonometric ratio to find the apothem *a*.
$a = \frac{2.5}{\tan 30°}$	Solve for *a*.
≈ 4.3	Use a calculator.
$A = \frac{1}{2}Pa$	Area of a regular polygon
$\approx \frac{1}{2}(30)(4.3)$	Replace *P* with 30 and *a* with 4.3.
≈ 64.5	Multiply.

So, the area of the base B is approximately 64.5 square centimeters.

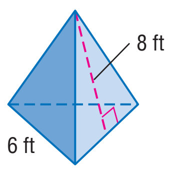
Step 3 Find the surface area of the pyramid.

$S = \frac{1}{2}P\ell + 1$	Surface area of a regular pyramid
$= \frac{1}{2}(30)(8) + 64.5$	$P = 30$, $\ell = 8$, and $B \approx 64.5$
≈ 184.5	Simplify.

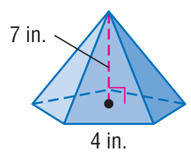
The surface area of the pyramid is about 184.5 square centimeters.

GuidedPractice

3A.



3B.



ReviewVocabulary

Trigonometric Ratios

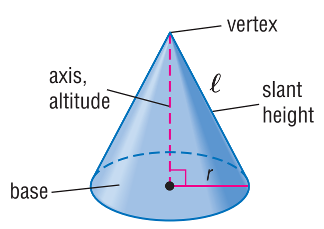
$\sin A = \frac{\text{opp}}{\text{hyp}}$

$\cos A = \frac{\text{adj}}{\text{hyp}}$

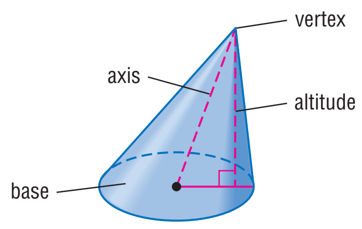
$\tan A = \frac{\text{opp}}{\text{adj}}$

2 Lateral Area and Surface Area of Cones

Recall that a cone has a circular base and a vertex. The axis of a cone is the segment with endpoints at the vertex and the center of the base. If the axis is also the altitude, then the cone is a **right cone**. If the axis is not the altitude, then the cone is an **oblique cone**.

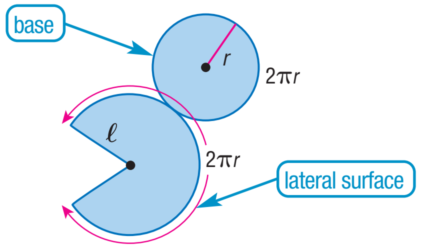


Right Cone



Oblique Cone

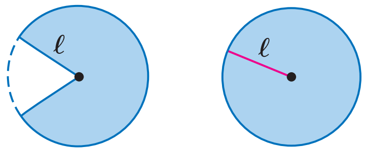
The net for a cone is shown at the right. The circle with radius r is the base of the cone. It has a circumference of $2\pi r$ and an area of πr^2. The sector with radius ℓ is the lateral surface of the cone. Its arc measure is $2\pi r$. You can use a proportion to find its area.



$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{measure of arc}}{\text{circumference of circle}}$$

$$\frac{\text{area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell}$$

$$\text{area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell} \text{ or } \pi r \ell$$



StudyTip

CCSS Sense-Making Like a pyramid, the lateral area of a right circular cone L equals $\frac{1}{2}P\ell$. Since the base is a circle, the perimeter is the circumference of the base C. So, the lateral area is $\frac{1}{2}C\ell$.

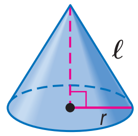
$$L = \frac{1}{2}C\ell$$
$$= \frac{1}{2}(2\pi r)\ell$$
$$= \pi r \ell$$

KeyConcept Lateral and Surface Area of a Cone

Words The lateral area L of a right circular cone is $L = \pi r \ell$, where r is the radius of the base and ℓ is the slant height.

The surface area S of a right circular cone is $S = \pi r \ell + \pi r^2$, where r is the radius of the base and ℓ is the slant height.

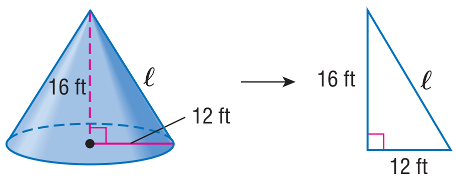
Model



Symbols $L = \pi r \ell$ $S = \pi r \ell + \pi r^2$

Real-World Example 4 Lateral Area of a Cone

ARCHITECTURE The conical slate roof at the right has a height of 16 feet and a radius of 12 feet. Find the lateral area.



Step 1 Find the slant height ℓ.

$\ell^2 = 16^2 + 12^2$ Pythagorean Theorem

$\ell^2 = 400$ Simplify.

$\ell = 20$ Take the positive square root of each side.

Step 2 Find the lateral area L.

Estimate $L \approx 3 \cdot 12 \cdot 20$ or 720 ft^2

$L = \pi r \ell$ Lateral area of a cone

$= \pi(12)(20)$ $r = 12$ and $\ell = 20$

≈ 754 Use a calculator.

The lateral area of the conical roof is about 754 square feet. The answer is reasonable compared to the estimate.

StudyTip

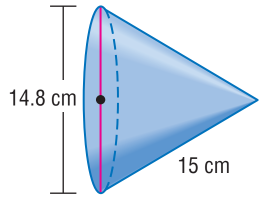
Draw a Diagram When solving word problems involving solids, it is helpful to draw a figure and label the known parts. Use a variable to label the measure or measures that you need to find.

GuidedPractice

4. ICE CREAM A waffle cone is $5\frac{1}{2}$ inches tall and the diameter of the base is $2\frac{1}{2}$ inches. Find the lateral area of the cone. Round to the nearest tenth.

Example 5 Surface Area of a Cone

Find the surface area of a cone with a diameter of 14.8 centimeters and a slant height of 15 centimeters.



Estimate: $S \approx 3 \cdot 7 \cdot 20 + 3 \cdot 50$ or $570\ \text{cm}^2$

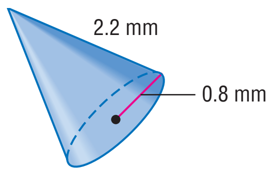
$S = \pi r\ell + \pi r^2$	Surface area of a cone
$= \pi(7.4)(15) + \pi(7.4)^2$	$r = 7.4$ and $\ell = 15$
≈ 520.8	Use a calculator.

The surface area of the cone is about 520.8 square centimeters. This is close to the estimate, so the answer is reasonable.

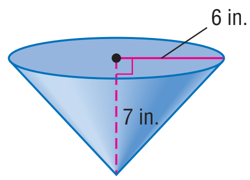
GuidedPractice

Find the surface area of each cone. Round to the nearest tenth.

5A.

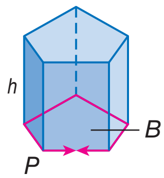
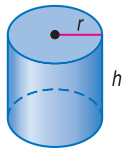
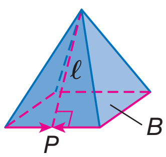
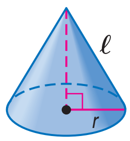


5B.



The formulas for lateral and surface area are summarized below.

ConceptSummary Lateral and Surface Areas of Solids

Solid	Model	Lateral Area	Surface Area
prism		$L = Ph$	$S = L + 2B$ or $S = Ph + 2B$
cylinder		$L = 2\pi rh$	$S = L + 2B$ or $S = 2\pi rh + 2\pi r^2$
pyramid		$L = \frac{1}{2}P\ell$	$S = \frac{1}{2}P\ell + B$
cone		$L = \pi r\ell$	$S = \pi r\ell + \pi r^2$

WatchOut!

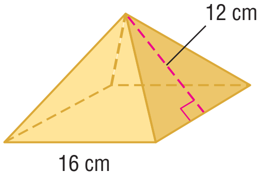
Bases The bases of right prisms and right pyramids are not always regular polygons.

Check Your Understanding

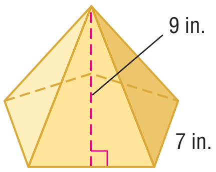
● = Step-by-Step Solutions begin on page R14.

Examples 1–3 **Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth if necessary.**

1.



2.



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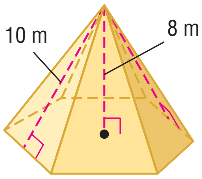
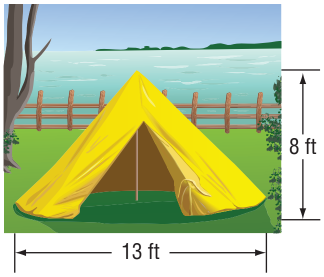


Examples 4–5 **4. TENTS** A conical tent is shown at the right. Round answers to the nearest tenth.

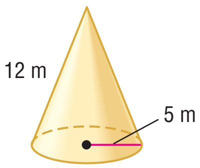
a. Find the lateral area of the tent and describe what it represents.

b. Find the surface area of the tent and describe what it represents.

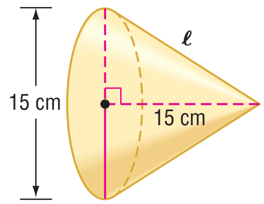


CCSS SENSE-MAKING **Find the lateral area and surface area of each cone. Round to the nearest tenth.**

5.



6.

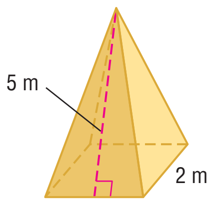


Practice and Problem Solving

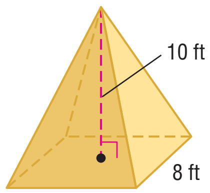
Extra Practice is on page R12.

Examples 1–3 **Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth if necessary.**

7



8.



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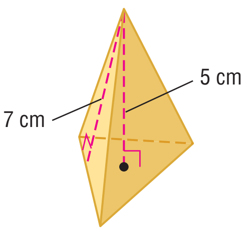
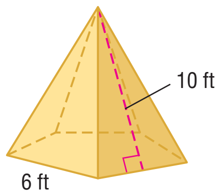


10.



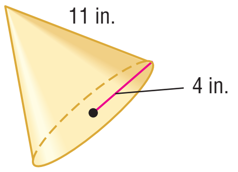
11. square pyramid with an altitude of 12 inches and a slant height of 18 inches

12. hexagonal pyramid with a base edge of 6 millimeters and a slant height of 9 millimeters

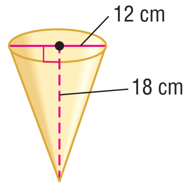
13. ARCHITECTURE Find the lateral area of a pyramid-shaped building that has a slant height of 210 feet and a square base 332 feet by 332 feet.

Examples 4–5 **Find the lateral area and surface area of each cone. Round to the nearest tenth.**

14.



15.



16. The diameter is 3.4 centimeters, and the slant height is 6.5 centimeters.

17. The altitude is 5 feet, and the slant height is $9\frac{1}{2}$ feet.

18. **MOUNTAINS** A conical mountain has a radius of 1.6 kilometers and a height of 0.5 kilometer. What is the lateral area of the mountain?

19 **HISTORY** Archaeologists recently discovered a 1500-year-old pyramid in Mexico City. The square pyramid measures 165 yards on each side and once stood 20 yards tall. What was the original lateral area of the pyramid?

20. Describe two polyhedrons that have 7 faces.

21. What is the sum of the number of faces, vertices, and edges of an octagonal pyramid?

22. **TEPEES** The dimensions of two canvas tepees are shown in the table at the right. Not including the floors, approximately how much more canvas is used to make Tepee B than Tepee A?

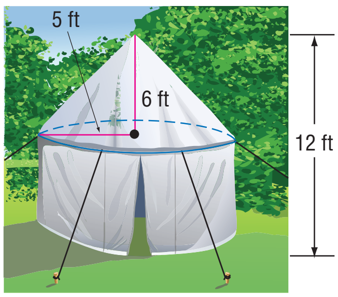
Tepee	Diameter (ft)	Height (ft)
A	14	6
B	20	9

23. The surface area of a square pyramid is 24 square millimeters and the base area is 4 square millimeters. What is the slant height of the pyramid?

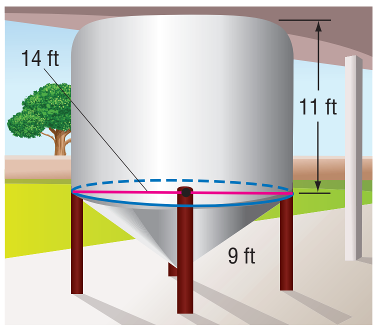
24. The surface area of a cone is 18π square inches and the radius of the base is 3 inches. What is the slant height of the cone?

25. The surface area of a triangular pyramid is 532 square centimeters, and the base is 24 centimeters wide with a hypotenuse of 25 centimeters. What is the slant height of the pyramid?

26. Find the lateral area of the tent to the nearest tenth.



27. Find the surface area of the tank. Write in terms of π.



28. **CHANGING DIMENSIONS** A cone has a radius of 6 centimeters and a slant height of 12 centimeters. Describe how each change affects the surface area of the cone.

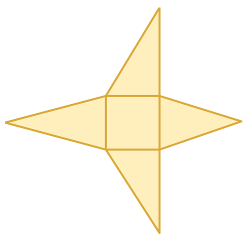
a. The radius and the slant height are doubled.

b. The radius and the slant height are divided by 3.

29. CCSS **TOOLS** A solid has the net shown at the right.

a. Describe the solid.

b. Make a sketch of the solid.

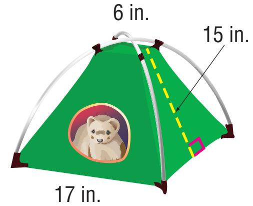


Sketch each solid and a net that represents the solid.

30. hexagonal pyramid

31. rectangular pyramid

32. **PETS** A *frustum* is the part of a solid that remains after the top portion has been cut by a plane parallel to the base. The ferret tent shown at the right is a frustum of a regular pyramid.



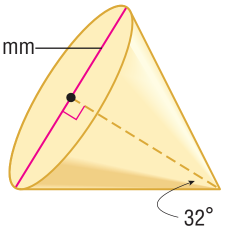
a. Describe the faces of the solid.

b. Find the lateral area and surface area of the frustum formed by the tent.

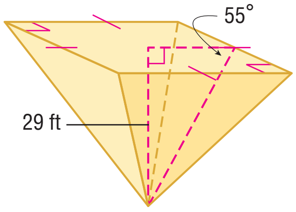
c. Another pet tent is made by cutting the top half off of a pyramid with a height of 12 centimeters, slant height of 20 centimeters and square base with side lengths of 32 centimeters. Find the surface area of the frustum.

Find the lateral area and surface area of each solid. Round to the nearest tenth.

33



34.



35. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the lateral and surface area of a square pyramid with a base edge of 3 units.

a. **Geometric** Sketch the pyramid on isometric dot paper.

b. **Tabular** Make a table showing the lateral areas of the pyramid for slant heights of 1, 3, and 9 units.

c. **Verbal** Describe what happens to the lateral area of the pyramid if the slant height is tripled.

d. **Analytical** Make a conjecture about how the lateral area of a square pyramid is affected if both the slant height and the base edge are tripled. Then test your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

36. **WRITING IN MATH** Why does an oblique solid not have a slant height?

37. **REASONING** Classify the following statement as *sometimes*, *always*, or *never* true. Justify your reasoning.

The surface area of a cone of radius r and height h is less than the surface area of a cylinder of radius r and height h.

38. **REASONING** A cone and a square pyramid have the same surface area. If the areas of their bases are also equal, do they have the same slant height as well? Explain.

39. **OPEN ENDED** Describe a pyramid that has a total surface area of 100 square units.

40. CCSS **ARGUMENTS** Determine whether the following statement is *true* or *false*. Explain your reasoning.

A regular polygonal pyramid and a cone both have height h units and base perimeter P units. Therefore, they have the same total surface area.

41. **WRITING IN MATH** Describe how to find the surface area of a regular polygonal pyramid with an *n*-gon base, height *h* units, and an apothem of *a* units.

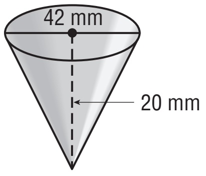
Standardized Test Practice

42. The top of a gazebo in a park is in the shape of a regular pentagonal pyramid. Each side of the pentagon is 10 feet long. If the slant height of the roof is about 6.9 feet, what is the lateral roof area?

A 34.5 ft^2 **C** 172.5 ft^2

B 50 ft^2 **D** 250 ft^2

43. SHORT RESPONSE To the nearest square millimeter, what is the surface area of a cone with the dimensions shown?



44. ALGEBRA Yu-Jun's craft store sells 3 handmade barrettes for $9.99. Which expression can be used to find the total cost C of x barrettes?

F $C = \frac{9.99}{x}$ **H** $C = 3.33x$

G $C = 9.99x$ **J** $C = \frac{x}{3.33}$

45. SAT/ACT What is the slope of a line perpendicular to the line with equation $2x + 3y = 9$?

A $-\frac{3}{2}$ **D** $\frac{3}{2}$

B $-\frac{2}{3}$ **E** $\frac{9}{2}$

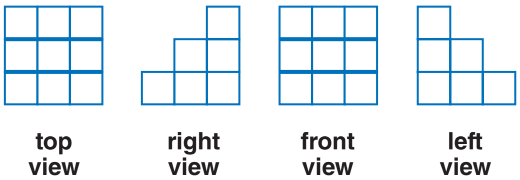
C $\frac{2}{3}$

Spiral Review

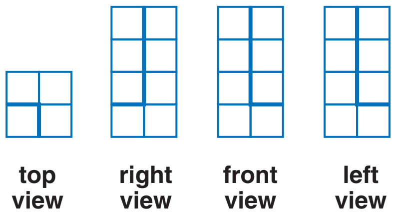
46. Find the surface area of a cylinder with a diameter of 18 cm and a height of 12 cm. (Lesson 12-2)

Use isometric dot paper and each orthographic drawing to sketch a solid. (Lesson 12-1)

47.



48.



Graph each figure and its image in the given line. (Lesson 9-1)

$J(2, 4), K(4, 0), L(7, 3)$

49. $\triangle JKL; x = 2$

50. $\triangle JKL; y = 1$

$Q(4, 8), R(1, 6), S(2, 1), T(5, 5)$

51. $QRST; y = -1$

52. $QRST; x = 4$

$A(-2, 6), B(-2, 1), C(3, 1), D(3, 4)$

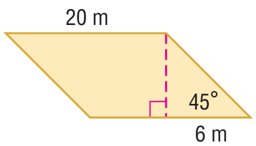
53. $ABCD; x = 1$

54. $ABCD; y = -2$

Skills Review

Find the perimeter and area of each parallelogram, triangle, or composite figure. Round to the nearest tenth.

55.



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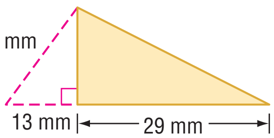

57.