LESSON 12-5

Volumes of Pyramids and Cones

Then

- You found surface areas of pyramids and cones.

Now

1. Find volumes of pyramids.
2. Find volumes of cones.

Why?

- Marta is studying crystals that grow on rock formations. For a project, she is making a clay model of a crystal with a shape that is a composite of two congruent rectangular pyramids. The base of each pyramid will be 1 by 1.5 inches, and the total height will be 4 inches. Why is determining the volume of the model helpful in this situation?

Gary Ombler/Dorling Kindersley/Getty Images

Common Core State Standards

Content Standards

G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

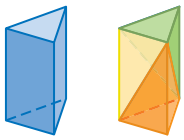
Mathematical Practices

1 Make sense of problems and persevere in solving them.

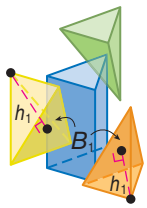
7 Look for and make use of structure.

1 Volume of Pyramids

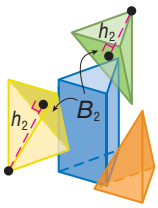
A triangular prism can be separated into three triangular pyramids as shown. Since all faces of a triangular pyramid are triangles, any face can be considered a base of the pyramid.



The yellow and orange pyramids have base area B_1 and height h_1. Therefore, by Cavalieri's Principle, they have the same volume. Likewise, the yellow and green pyramids have base area B_2 and height h_2, so they have the same volume.

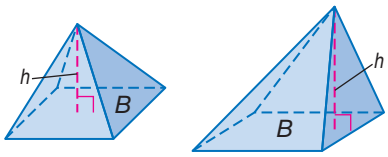


Since the orange and green pyramids have the same volume as the yellow pyramid, it follows that the volumes of all three pyramids are the same. Therefore, each pyramid has one third the volume of the prism with the same base area and height. This is true for a pyramid with any shape base.



KeyConcept Volume of a Pyramid

Words	The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid.	**Models**
Symbols	$V = \frac{1}{3}Bh$	



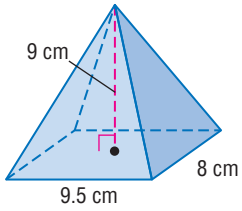
Example 1 Volume of a Pyramid

Find the volume of the pyramid.

$V = \frac{1}{3}Bh$ Volume of a pyramid

$= \frac{1}{3}(9.5 \cdot 8)(9)$ $B = 9.5 \cdot 8$ and $h = 9$

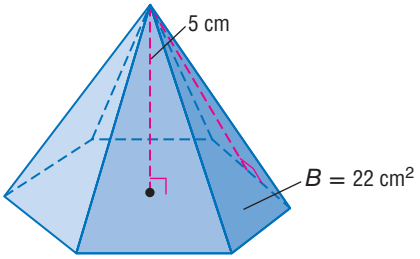
$= 228$ Simplify.



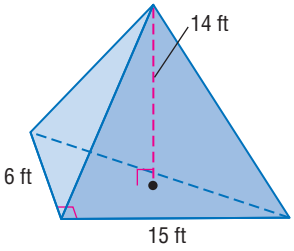
The volume of the pyramid is 228 cubic centimeters.

GuidedPractice

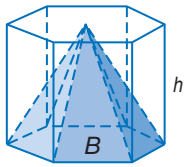
1A.

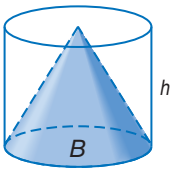


1B.



2 Volume of Cones The pyramid and prism shown have the same base area B and height h as the cylinder and cone. Since the volume of the pyramid is one third the volume of the prism, then by Cavalieri's Principle, the volume of the cone must be one third the volume of the cylinder.





WatchOut!

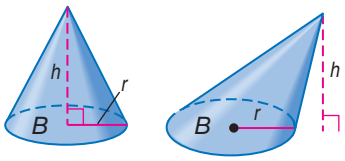
Volumes of Cones
The formula for the surface area of a cone only applies to right cones. However, the formula for volume applies to oblique cones as well as right cones.

KeyConcept Volume of a Cone

Words The volume of a circular cone is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$, where B is the area of the base, h is the height of the cone, and r is the radius of the base.

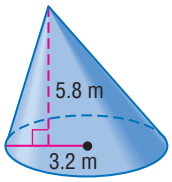
Symbols $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$

Models



Example 2 Volume of a Cone

a. Find the volume of the cone. Round to the nearest tenth.



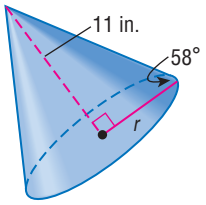
$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$\approx \frac{1}{3}\pi(3.2)^2(5.8)$ $r = 3.2$ and $h = 5.8$

≈ 62.2 Use a calculator.

The volume of the cone is approximately 62.2 cubic meters.

b. Find the volume of the cone. Round to the nearest tenth.



Step 1 Use trigonometry to find the radius.

$\tan 58° = \frac{11}{r}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

$r = \frac{11}{\tan 58°}$ Solve for r.

$r \approx 6.9$ Use a calculator.

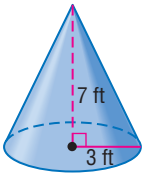
Step 2 Find the volume.

$$V = \frac{1}{3}\pi r^2 h \qquad \text{Volume of a cone}$$
$$\approx \frac{1}{3}\pi(6.9)^2(11) \qquad r \approx 6.9 \text{ and } h = 11$$
$$\approx 548.4 \qquad \text{Use a calculator.}$$

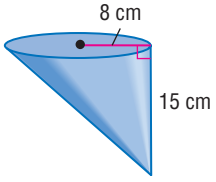
The volume of the cone is approximately 548.4 cubic inches.

GuidedPractice

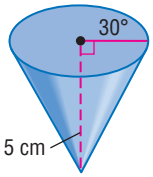
2A.



2B.



2C.



Real-WorldLink

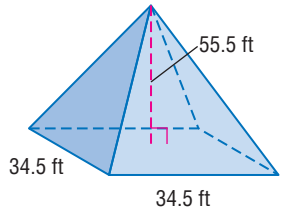
The Washington Monument is the largest masonry structure in the world. By law, no other building in D.C. is allowed to be taller than the 555-foot-tall structure.

Source: Enchanted Learning

Real-World Example 3 Find Real-World Volumes

ARCHITECTURE **At the top of the Washington Monument is a small square pyramid, called a *pyramidion*. This pyramid has a height of 55.5 feet with base edges of approximately 34.5 feet. What is the volume of the pyramidion? Round to the nearest tenth.**

Sketch and label the pyramid.



$$V = \frac{1}{3}Bh \qquad \text{Volume of a pyramid}$$
$$= \frac{1}{3}(34.5 \cdot 34.5)(55.5) \qquad B = 34.5 \cdot 34.5,\ h = 55.5$$
$$\approx 22{,}019.6 \qquad \text{Simplify.}$$

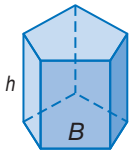
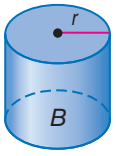
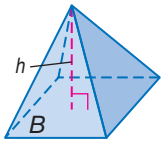
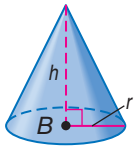
The volume of the pyramidion atop the Washington Monument is about 22,019.6 cubic feet.

GuidedPractice

3. ARCHAEOLOGY A pyramidion that was discovered in Saqqara, Egypt, in 1992 has a rectangular base 53 centimeters by 37 centimeters. It is 46 centimeters high. What is the volume of this pyramidion? Round to the nearest tenth.

The formulas for the volumes of solids are summarized below.

ConceptSummary Volumes of Solids

Solid	prism	cylinder	pyramid	cone
Model				
Volume	$V = Bh$	$V = Bh$ or $V = \pi r^2 h$	$V = \frac{1}{3}Bh$	$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$

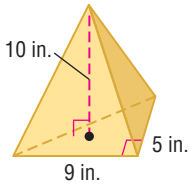
Jupiterimages/Brand X/Alamy

Check Your Understanding

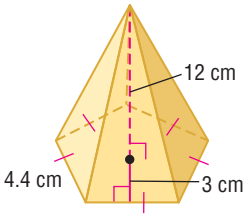
= Step-by-Step Solutions begin on page R14.

Example 1 **Find the volume of each pyramid.**

1.



2.

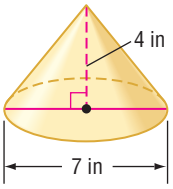


3. a rectangular pyramid with a height of 5.2 meters and a base 8 meters by 4.5 meters

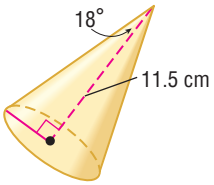
4. a square pyramid with a height of 14 meters and a base with 8-meter side lengths

Example 2 **Find the volume of each cone. Round to the nearest tenth.**

5.



6.



7. an oblique cone with a height of 10.5 millimeters and a radius of 1.6 millimeters

8. a cone with a slant height of 25 meters and a radius of 15 meters

Example 3

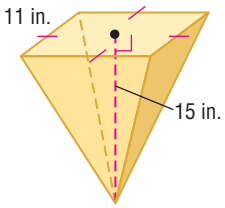
9. **MUSEUMS** The sky dome of the National Corvette Museum in Bowling Green, Kentucky, is a conical building. If the height is 100 feet and the area of the base is about 15,400 square feet, find the volume of air that the heating and cooling systems would have to accommodate. Round to the nearest tenth.

Practice and Problem Solving

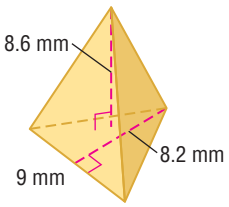
Extra Practice is on page R12.

Example 1 **CCSS SENSE-MAKING** **Find the volume of each pyramid. Round to the nearest tenth if necessary.**

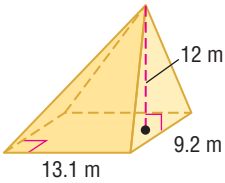
10.



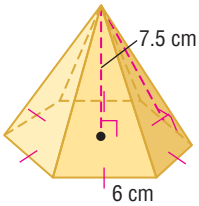
11



12.



13.



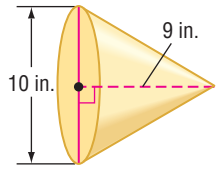
14. a pentagonal pyramid with a base area of 590 square feet and an altitude of 7 feet

15. a triangular pyramid with a height of 4.8 centimeters and a right triangle base with a leg 5 centimeters and hypotenuse 10.2 centimeters

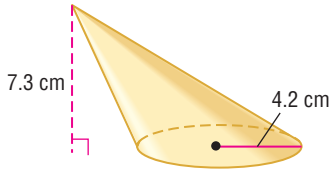
16. A triangular pyramid with a right triangle base with a leg 8 centimeters and hypotenuse 10 centimeters has a volume of 144 cubic centimeters. Find the height.

Example 2 **Find the volume of each cone. Round to the nearest tenth.**

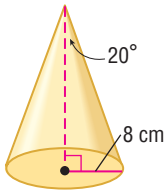
17.



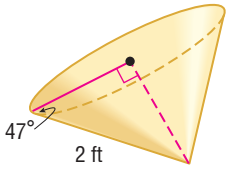
18.



19.



20.



21. an oblique cone with a diameter of 16 inches and an altitude of 16 inches

22. a right cone with a slant height of 5.6 centimeters and a radius of 1 centimeter

Example 3

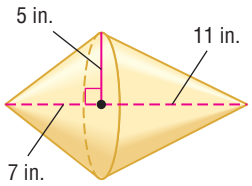
23. **SNACKS** Approximately how many cubic centimeters of roasted peanuts will completely fill a paper cone that is 14 centimeters high and has a base diameter of 8 centimeters? Round to the nearest tenth.

24. CCSS **MODELING** The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. It is approximately 350 feet tall, and its square base is 600 feet wide. Find the volume of this pyramid.

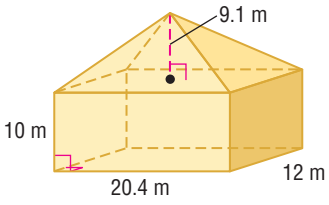
25. **GARDENING** The greenhouse at the right is a regular octagonal pyramid with a height of 5 feet. The base has side lengths of 2 feet. What is the volume of the greenhouse?

Find the volume of each solid. Round to the nearest tenth.

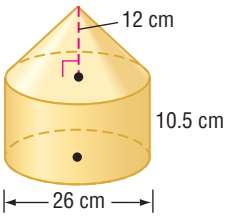
26.



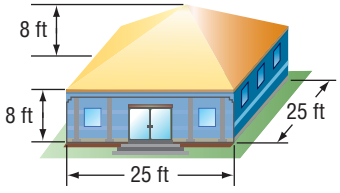
27.



28.



29. **HEATING** Sam is building an art studio in her backyard. To buy a heating unit for the space, she needs to determine the BTUs (British Thermal Units) required to heat the building. For new construction with good insulation, there should be 2 BTUs per cubic foot. What size unit does Sam need to purchase?



30. **SCIENCE** Refer to page 873. Determine the volume of the model. Explain why knowing the volume is helpful in this situation.

31. CHANGING DIMENSIONS A cone has a radius of 4 centimeters and a height of 9 centimeters. Describe how each change affects the volume of the cone.

a. The height is doubled.

b. The radius is doubled.

c. Both the radius and the height are doubled.

Find each measure. Round to the nearest tenth if necessary.

32. A square pyramid has a volume of 862.5 cubic centimeters and a height of 11.5 centimeters. Find the side length of the base.

33 The volume of a cone is 196π cubic inches and the height is 12 inches. What is the diameter?

34. The lateral area of a cone is 71.6 square millimeters and the slant height is 6 millimeters. What is the volume of the cone?

35. MULTIPLE REPRESENTATIONS In this problem, you will investigate rectangular pyramids.

a. Geometric Draw two pyramids with different bases that have a height of 10 centimeters and a base area of 24 square centimeters.

b. Verbal What is true about the volumes of the two pyramids that you drew? Explain.

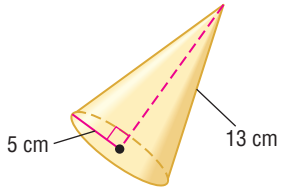
c. Analytical Explain how multiplying the base area and/or the height of the pyramid by 5 affects the volume of the pyramid.

H.O.T. Problems Use Higher-Order Thinking Skills

36. CCSS ARGUMENTS Determine whether the following statement is *always*, *sometimes*, or *never* true. Justify your reasoning.

The volume of a cone with radius r and height h equals the volume of a prism with height h.

37. ERROR ANALYSIS Alexandra and Cornelio are calculating the volume of the cone at the right. Is either of them correct? Explain your answer.



Alexandra

$V = \frac{1}{3}Bh$

$= \frac{1}{3}\pi(5^2)(13)$

$\approx 340.3 \text{ cm}^3$

Cornelio

$5^2 + 12^2 = 13^2$

$V = \frac{1}{3}Bh$

$= \frac{1}{3}\pi(5^2)(12)$

$\approx 314.2 \text{ cm}^3$

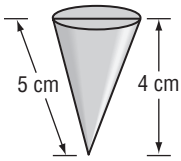
38. REASONING A cone has a volume of 568 cubic centimeters. What is the volume of a cylinder that has the same radius and height as the cone? Explain your reasoning.

39. OPEN ENDED Give an example of a pyramid and a prism that have the same base and the same volume. Explain your reasoning.

40. WRITING IN MATH Compare and contrast finding volumes of pyramids and cones with finding volumes of prisms and cylinders.

Standardized Test Practice

41. A conical sand toy has the dimensions as shown below. How many cubic centimeters of sand will it hold when it is filled to the top?



A 12π

B 15π

C $\frac{80}{3}\pi$

D $\frac{100}{3}\pi$

42. SHORT RESPONSE Brooke is buying a tent that is in the shape of a rectangular pyramid. The base is 6 feet by 8 feet. If the tent holds 88 cubic feet of air, how tall is the tent's center pole?

43. PROBABILITY A spinner has sections colored red, blue, orange, and green. The table below shows the results of several spins. What is the experimental probability of the spinner landing on orange?

F $\frac{1}{5}$

G $\frac{1}{4}$

H $\frac{9}{25}$

J $\frac{1}{2}$

Color	Frequency
red	6
blue	4
orange	5
green	10

44. SAT/ACT For all $x \neq -2$ or 0, $\frac{x^2 - 2x - 8}{x^2 + 2x} = ?$

A -8

B $x - 4$

C $\frac{-x - 4}{x}$

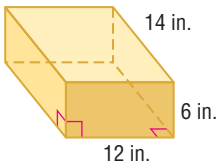
D $\frac{-8}{x + 2}$

E $\frac{x - 4}{x}$

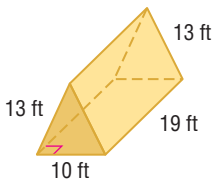
Spiral Review

Find the volume of each prism. (Lesson 12-4)

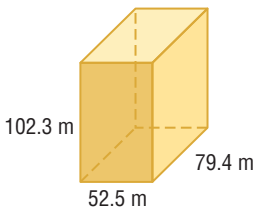
45.



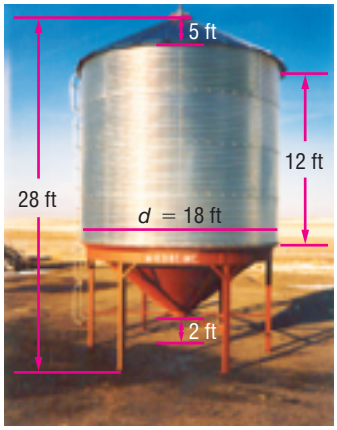
46.



47.



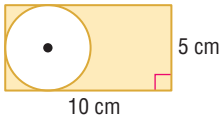
48. FARMING The picture shows a combination hopper cone and bin used by farmers to store grain after harvest. The cone at the bottom of the bin allows the grain to be emptied more easily. Use the dimensions in the diagram to find the entire surface area of the bin with a conical top and bottom. Write the exact answer and the answer rounded to the nearest square foot. (Lesson 12-3)



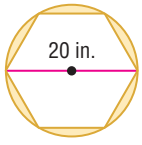
Skills Review

Find the area of each shaded region. The polygons in Exercises 50-52 are regular.

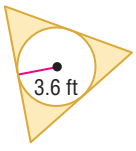
49.



50.



51.



52.