CHAPTER 2

Study Guide and Review

Study Guide

KeyConcepts

Inductive Reasoning and Logic (Lessons 2-1 and 2-2)

- Inductive reasoning: a conjecture is reached based on observations of a previous pattern
- Counterexample: an example that proves a conjecture is false
- Negation of statement p: *not* p
- Conjunction: a compound statement formed with the word *and*
- Disjunction: a compound statement formed with the word *or*

Conditional Statements (Lesson 2-3)

- An if-then statement is written in the form if p, then q in which p is the hypothesis and q is the conclusion.

statement	$p \rightarrow q$
converse	$q \rightarrow p$
inverse	not $p \rightarrow$ not q
contrapositive	not $q \rightarrow$ not p

Deductive Reasoning (Lesson 2-4)

- Law of Detachment: If $p \rightarrow q$ is true and p is true, then q is also true.
- Law of Syllogism: If $p \rightarrow q$ and $q \rightarrow r$ are true, then $p \rightarrow r$ is also true.

Proof (Lessons 2-5 through 2-8)

Step 1 List the given information and draw a diagram, if possible.

Step 2 State what is to be proved.

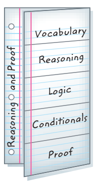
Step 3 Create a deductive argument.

Step 4 Justify each statement with a reason.

Step 5 State what you have proved.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

algebraic proof (p. 136)
axiom (p. 127)
compound statement (p. 99)
conclusion (p. 107)
conditional statement (p. 107)
conjecture (p. 91)
conjunction (p. 99)
contrapositive (p. 109)
converse (p. 109)
counterexample (p. 94)
deductive argument (p. 129)
deductive reasoning (p. 117)
disjunction (p. 100)
formal proof (p. 137)
hypothesis (p. 107)
if-then statement (p. 107)
inductive reasoning (p. 91)
informal proof (p. 129)
inverse (p. 109)
logically equivalent (p. 110)
negation (p. 99)
paragraph proof (p. 129)
postulate (p. 127)
proof (p. 128)
related conditionals (p. 109)
statement (p. 99)
theorem (p. 129)
truth table (p. 101)
truth value (p. 99)
two-column proof (p. 137)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. A <u>postulate</u> is a statement that requires proof.
2. The first part of an if-then statement is the <u>conjecture</u>.
3. <u>Deductive reasoning</u> uses the laws of mathematics to reach logical conclusions from given statements.
4. The <u>contrapositive</u> is formed by negating the hypothesis and conclusion of a conditional.
5. A <u>conjunction</u> is formed by joining two or more statements with the word *and*.
6. A <u>theorem</u> is a statement that is accepted as true without proof.
7. The <u>converse</u> is formed by exchanging the hypothesis and conclusion of a conditional.
8. To show that a conjecture is false, you would provide a <u>disjunction</u>.
9. The <u>inverse</u> of a statement p would be written in the form *not p*.
10. In a two-column proof, the properties that justify each step are called <u>reasons</u>.

Lesson-by-Lesson Review

2-1 Inductive Reasoning and Conjecture

Determine whether each conjecture is *true* or *false*. If false, give a counterexample.

11. If $\angle 1$ and $\angle 2$ are supplementary angles, then $\angle 1$ and $\angle 2$ form a linear pair.

12. If $W(-3, 2)$, $X(-3, 7)$, $Y(6, 7)$, $Z(6, 2)$, then quadrilateral $WXYZ$ is a rectangle.

13. **PARKS** Jacinto enjoys hiking with his dog in the forest at his local park. While on vacation in Smoky Mountain National Park in Tennessee, he was disappointed that dogs were not allowed on most hiking trails. Make a conjecture about why his local park and the national park have differing rules with regard to pets.

Example 1

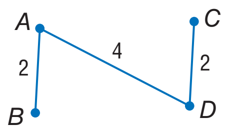
Determine whether each conjecture is *true* or *false*. If false, give a counterexample.

a. $c = d$, $d = c$ **is an example of a property of real numbers.**

$c = d$, $d = c$ is an example of the Symmetric Property of real numbers, so the conjecture is true.

b. **If** $AB + CD = AD$, **then** B **and** C **are between** A **and** D.

This conjecture is false. In the figure below, $AB + CD = AD$, but B and C are not between A and D.



2-2 Logic

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain.

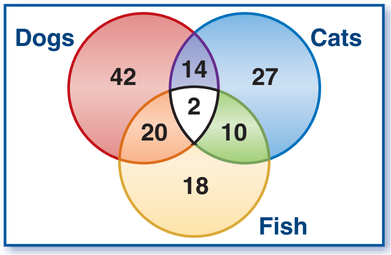
p: A plane contains at least three noncollinear points.

q: A square yard is equivalent to three square feet.

r: The sum of the measures of two complementary angles is 180.

14. $\sim q \vee r$ **15.** $p \wedge \sim r$ **16.** $\sim p \vee q$

17. **PETS** The Venn diagram shows the results of a pet store survey to determine the pets customers owned.



a. How many customers had only fish?

b. How many had only cats and dogs?

c. How many had dogs as well as fish?

Example 2

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain.

p: x^2 is a nonnegative number.

q: Adjacent angles lie in the same plane.

r: A negative number is not a real number.

a. $\sim q \wedge r$

$\sim q \wedge r$: Adjacent angles do not lie in the same plane, and a negative number is not a real number.

Since both $\sim q$ and r are false, $\sim q \wedge r$ is false.

b. p or r

p or r: x^2 is a nonnegative number, or a negative number is not a real number.

p or r is true because p is true. It does not matter that r is false.

2-3 Conditional Statements

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

18. If you square an integer, then the result is a positive integer.

19. If a hexagon has eight sides, then all of its angles will be obtuse.

20. Write the converse, inverse, and contrapositive of the following true conditional. Then, determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

If two angles are congruent, then they have the same degree measure.

Example 3

Write the *converse*, *inverse*, and *contrapositive* of the following true conditional.

If a figure is a square, then it is a parallelogram.

Converse: If a figure is a parallelogram, then it is a square.

Inverse: If a figure is not a square, then it is not a parallelogram.

Contrapositive: If a figure is not a parallelogram, then it is not a square.

2-4 Deductive Reasoning

Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

21. Given: If a quadrilateral has diagonals that bisect each other, then it is a parallelogram.

The diagonals of quadrilateral *PQRS* bisect each other.

22. Given: If Liana struggles in science class, then she will receive tutoring.

If Liana stays after school on Thursday, then she will receive tutoring.

23. **EARTHQUAKES** Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain.

Given: If an earthquake measures a 7.0 or higher on the Richter scale, then it is considered a major earthquake that could cause serious damage. The 1906 San Francisco earthquake measured 8.0 on the Richter scale.

Conclusion: The 1906 San Francisco earthquake was a major earthquake that caused serious damage.

Example 4

Use the Law of Syllogism to determine whether a valid conclusion can be reached from the following statements.

(1) If the measure of an angle is greater than 90, then it is an obtuse angle.

(2) If an angle is an obtuse angle, then it is not a right angle.

p: the measure of an angle is greater than 90

q: the angle is an obtuse angle

r: the angle is not a right angle

Statement (1): $p \rightarrow q$

Statement (2): $q \rightarrow r$

Since the given statements are true, use the Law of Syllogism to conclude that $p \rightarrow r$. That is, *If the measure of an angle is greater than 90, then it is not a right angle.*

2-5 Postulates and Paragraph Proofs

Determine whether each statement is *always, sometimes,* or *never* true. Explain.

24. Two planes intersect at a point.

25. Three points are contained in more than one plane.

26. If line *m* lies in plane $\mathcal{X}$ and line *m* contains a point *Q*, then point *Q* lies in plane $\mathcal{X}$.

27. If two angles are complementary, then they form a right angle.

28. **NETWORKING** Six people are introduced at a business convention. If each person shakes hands with each of the others, how many handshakes will be exchanged? Include a model to support your reasoning.

Example 5

Determine whether each statement is *always, sometimes,* or *never* true. Explain.

a. ***If points X, Y, and Z lie in plane $\mathcal{R}$, then they are not collinear.***

Sometimes; the fact that *X, Y,* and *Z* are contained in plane $\mathcal{R}$ has no bearing on whether those points are collinear or not.

b. **For any two points *A* and *B*, there is exactly one line that contains them.**

Always; according to Postulate 2-1, there is exactly one line through any two points.

2-6 Algebraic Proof

State the property that justifies each statement.

29. If $7(x-3)=35$, then $35=7(x-3)$.

30. If $2x+19=27$, then $2x=8$.

31. $5(3x+1)=15x+5$

32. $7x-2=7x-2$

33. If $12=2x+8$ and $2x+8=3y$, then $12=3y$.

34. Copy and complete the following proof.

Given: $6(x-4)=42$

Prove: $x=11$

Statements	Reasons
a. $6(x-4)=42$	**a.** ?
b. $6x-24=42$	**b.** ?
c. $6x=66$	**c.** ?
d. $x=11$	**d.** ?

35. Write a two-column proof to show that if $PQ=RS$, $PQ=5x+9$, and $RS=x-31$, then $x=-10$.

36. **GRADES** Jerome received the same quarter grade as Paula. Paula received the same quarter grade as Heath. Which property would show that Jerome and Heath received the same grade?

Example 6

Write a two-column proof.

Given: $\frac{5x-3}{6}=2x+1$

Prove: $x=-\frac{9}{7}$

Proof:

Statements	Reasons
1. $\frac{5x-3}{6}=2x+1$	**1.** Given
2. $5x-3=6(2x+1)$	**2.** Multiplication Property of Equality
3. $5x-3=12x+6$	**3.** Distributive Property of Equality
4. $-3=7x+6$	**4.** Subtraction Property of Equality
5. $-9=7x$	**5.** Subtraction Property of Equality
6. $-\frac{9}{7}=x$	**6.** Division Property of Equality
7. $x=-\frac{9}{7}$	**7.** Symmetric Property of Equality

2-7 Proving Segment Relationships

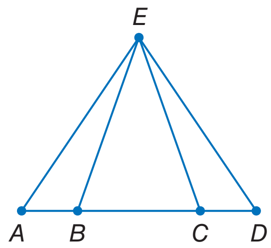
Write a two-column proof.

37. **Given:** X is the midpoint of $\overline{WY}$ and $\overline{VZ}$.

Prove: $VW = ZY$

38. **Given:** $AB = DC$

Prove: $AC = DB$



39. **GEOGRAPHY** Leandro is planning to drive from Kansas City to Minneapolis along Interstate 35. The map he is using gives the distance from Kansas City to Des Moines as 194 miles and from Des Moines to Minneapolis as 243 miles. What allows him to conclude that the distance he will be driving is 437 miles from Kansas City to Minneapolis? Assume that Interstate 35 forms a straight line.

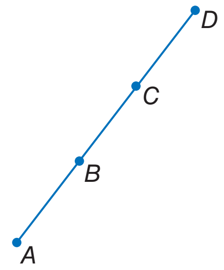
Example 7

Write a two-column proof.

Given: B is the midpoint of $\overline{AC}$.

C is the midpoint of $\overline{BD}$.

Prove: $\overline{AB} \cong \overline{CD}$



Proof:

Statements	Reasons
1. B is the midpoint of $\overline{AC}$.	**1.** Given
2. $\overline{AB} \cong \overline{BC}$	**2.** Definition of midpoint
3. C is the midpoint of $\overline{BD}$.	**3.** Given
4. $\overline{BC} \cong \overline{CD}$	**4.** Definition of midpoint
5. $\overline{AB} \cong \overline{CD}$	**5.** Transitive Property of Equality

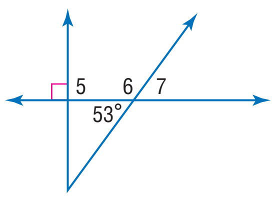
2-8 Proving Angle Relationships

Find the measure of each angle.

40. $\angle 5$

41. $\angle 6$

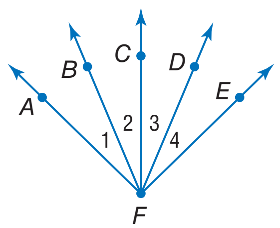
42. $\angle 7$



43. **PROOF** Write a two-column proof.

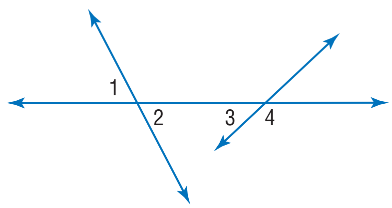
Given: $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$

Prove: $\angle AFC \cong \angle EFC$



Example 8

Find the measure of each numbered angle if $m\angle 1 = 72$ and $m\angle 3 = 26$.



$m\angle 2 = 72$, since $\angle 1$ and $\angle 2$ are vertical angles.

$\angle 3$ and $\angle 4$ form a linear pair and must be supplementary angles.

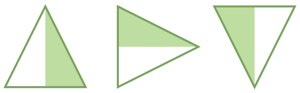
$26 + m\angle 4 = 180$ Definition of supplementary angles

$m\angle 4 = 154$ Subtract 26 from each side.

CHAPTER 2 Practice Test

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1. 15, 30, 45, 60

2. 

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

p: $5 < -3$

q: All vertical angles are congruent.

r: If $4x = 36$, then $x = 9$.

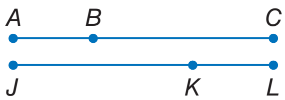
3. p and q

4. $(p \vee q) \wedge r$

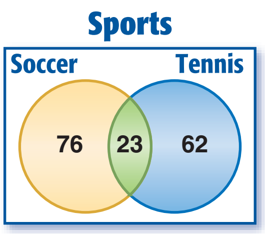
5. **PROOF** Write a paragraph proof.

Given: $\overline{JK} \cong \overline{CB}$, $\overline{KL} \cong \overline{AB}$

Prove: $\overline{JL} \cong \overline{AC}$



6. **SPORTS** Refer to the Venn diagram that represents the sports students chose to play at South High School last year.



a. Describe the sports that the students in the nonintersecting portion of the tennis region chose.

b. How many students played soccer and tennis?

7. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

Given: If a lawyer passes the bar exam, then he or she can practice law. Candice passed the bar exam.

Conclusion: Candice can practice law.

8. **PROOF** Copy and complete the following proof.

Given: $3(x - 4) = 2x + 7$

Prove: $x = 19$

Proof:

Statements	Reasons
a. $3(x - 4) = 2x + 7$	**a.** Given
b. $3x - 12 = 2x + 7$	**b.** ___?___
c. ___?___	**c.** Subtraction Property
d. $x = 19$	**d.** ___?___

Determine whether each statement is *always*, *sometimes*, or *never* true.

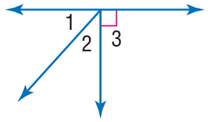
9. Two angles that are supplementary form a linear pair.

10. If B is between A and C, then $AC + AB = BC$.

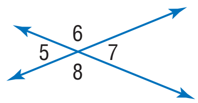
11. If two lines intersect to form congruent adjacent angles, then the lines are perpendicular.

Find the measure of each numbered angle, and name the theorems that justify your work.

12. $m\angle 1 = x$,
$m\angle 2 = x - 6$



13. $m\angle 7 = 2x + 15$,
$m\angle 8 = 3x$



Write each statement in if-then form.

14. An acute angle measures less than 90.

15. Two perpendicular lines intersect to form right angles.

16. **MULTIPLE CHOICE** If a triangle has one obtuse angle, then it is an obtuse triangle.

Which of the following statements is the contrapositive of the conditional above?

A If a triangle is not obtuse, then it has one obtuse angle.

B If a triangle does not have one obtuse angle, then it is not an obtuse triangle.

C If a triangle is not obtuse, then it does not have one obtuse angle.

D If a triangle is obtuse, then it has one obtuse angle.