## Study Guide and Review

## Study Guide

## KeyConcepts

Points, Lines, and Planes (Lesson 1-1)

- There is exactly one line through any two points.
- There is exactly one plane through any three noncollinear points.

Distance and Midpoints (Lesson 1-3)

- On a number line, the measure of a segment with endpoint coordinates $a$ and $b$ is $|a-b|$.
- In the coordinate plane, the distance between two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is given by $d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$.
- On a number line, the coordinate of the midpoint of a segment with endpoints $a$ and $b$ is $\frac{a+b}{2}$.
- In the coordinate plane, the coordinates of the midpoint of a segment with endpoints that are $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ are $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$.


## Angles (Lessons 1-4 and 1-5)

- An angle is formed by two noncollinear rays that have a common endpoint, called its vertex. Angles can be classified by their measures.
- Adjacent angles are two coplanar angles that lie in the same plane and have a common vertex and a common side but no common interior points.
- Vertical angles are two nonadjacent angles formed by two intersecting lines.
- A linear pair is a pair of adjacent angles with noncommon sides that are opposite rays.
- Complementary angles are two angles with measures that have a sum of 90 .
- Supplementary angles are two angles with measures that have a sum of 180 .


## FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.


## KeyVocabulary

acute angle (p. 38)
adjacent angles (p. 46)
angle (p. 36)
angle bisector (p. 39)
area (p. 58)
base (p. 67)
between (p. 15)
circumference (p. 58)
collinear (p. 5)
complementary angles (p. 47)
concave (p. 56)
cone (p. 67)
congruent (p. 16)
construction (p. 17)
convex (p. 56)
coplanar (p. 5)
cylinder (p.67)
degree (p. 37)
distance (p. 25)
edge (p. 67)
equiangular polygon (p. 57)
equilateral polygon (p. 57)
exterior (p. 36)
face (p. 67)
interior (p. 36)
intersection (p. 6)
line ( $\mathrm{p}, 5$ )
line segment (p. 14)
linear pair (p. 46)
midpoint (p. 27)
n-gon (p. 57)
obtuse angle (p. 38)
opposite rays (p. 36)
perimeter (p. 58)
perpendicular (p. 48)
plane (p. 5)
Platonic solid (p. 68)
point (p. 5)
polygon (p. 56)
polyhedron (p. 67)
prism (p. 67)
pyramid (p. 67)
ray (p. 36)
regular polygon (p. 57)
regular polyhedron (p.68)
right angle (p. 38)
segment bisector (p. 29)
side (p. 36)
space (p.7)
sphere (p. 67)
supplementary angles (p. 47)
surface area (p. 69)
undefined term (p. 5)
vertex (pp. 36, 67)
vertex of a polygon (p. 56)
vertical angles (p. 46)
volume (p.69)

## VocabularyCheck

Fill in the blank in each sentence with the vocabulary term that best completes the sentence.

1. A $\qquad$ is a flat surface made up of points that extends infinitely in all directions.
2. A set of points that all lie on the same line are said to be
$\qquad$ -.
3. If two lines intersect to form four right angles, the lines are
$\qquad$ -.
4. If the sum of the measures of two angles is 180, then the angles are called $\qquad$ angles.

## Lesson-by-Lesson Review

## Points, Lines, and Planes

Use the figure to complete each of the following.

5. Name the intersection of lines $a$ and $c$.
6. Give another name for line $b$.
7. Name a point that is not contained in any of the three lines $a, b$, or $c$.
8. Give another name for plane WPX.

Name the geometric term that is best modeled by each item.
9.

10.


## Example 1

Draw and label a figure for the relationship below.


Plane $X$ contains line $a$, line $b$ intersects line $a$ at point $Q$, but line $b$ is not in plane $X$.

Draw a surface to represent plane $\mathcal{X}$ and label it.
Draw a line in plane $\mathcal{X}$ and label it line $a$.
Draw a line $b$ intersecting both the plane and line $a$ and label the point of intersection $Q$.

## Linear Measure

Find the value of the variable and $X P$, if $X$ is between $P$ and $Q$.
11. $X Q=13, X P=5 x-3, P Q=40$
12. $X Q=3 k, X P=7 k-2, P Q=6 k+16$

Determine whether each pair of segments is congruent.
13. $\overline{A B}, \overline{C D}$

14. $\overline{X Y}, \overline{Y Z}$

15. DISTANCE The distance from Salvador's job to his house is 3 times greater than the distance from his house to school. If his house is between his job and school and the distance from his job to school is 6 miles, how far is it from Salvador's house to school?

## Example 2

Use the figure to find the value of the variable and the length of $\overline{Y Z}$.

$X Z=X Y$
$29=10+3 x+7$
Betweenness of points
$+7$
Substitution
$29=3 x+17$
Simplify.
$12=3 x$
$4=x$

$$
\begin{aligned}
Y Z & =3 x+7 \\
& =3(4)+7 \text { or } 19
\end{aligned}
$$

Subtract 17 from each side.
Divide each side by 3
Given
Substitution

So, $x=4$ and $Y Z=19$.

## Distance and Midipoints

Find the distance between each pair of points.
16. $A(-3,1), B(7,13)$
17. $P(2,-1), Q(10,-7)$

Find the coordinates of the midpoint of a segment with the given endpoints.
18. $L(-3,16), M(17,4)$
19. $C(32,-1), D(0,-12)$

Find the coordinates of the missing endpoint if $M$ is the midpoint of $\overline{X Y}$.
20. $X(-11,-6), M(15,4)$
21. $M(-4,8), Y(19,0)$
22. HIKING Carol and Marita are hiking in a state park and decide to take separate trails. The map of the park is set up on a coordinate grid. Carol's location is at the point $(7,13)$ and Marita is at $(3,5)$.
a. Find the distance between them.
b. Find the coordinates of the point midway between their locations.

## Example 3

Find the distance between $X(5,7)$ and $Y(-7,2)$.

$$
\begin{aligned}
& \text { Let }\left(x_{1}, y_{1}\right)=(5,7) \text { and }\left(x_{2}, y_{2}\right)=(-7,2) . \\
& \begin{aligned}
d & =\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& =\sqrt{(-7-5)^{2}+(2-7)^{2}} \\
& =\sqrt{(-12)^{2}+(-5)^{2}} \\
& =\sqrt{169} \text { or } 13
\end{aligned}
\end{aligned}
$$

The distance from $X$ to $Y$ is 13 units.

## Example 4

Find the coordinates of the midpoint between $P(-4,13)$ and $Q(6,5)$.
Let $\left(x_{1}, y_{1}\right)=(-4,13)$ and $\left(x_{2}, y_{2}\right)=(6,5)$.
$M\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)=M\left(\frac{-4+6}{2}, \frac{13+5}{2}\right)$

$$
=M(1,9)
$$

The coordinates of the midpoint are $(1,9)$.

## Angle Measure

For Exercises 23-26, refer to the figure below.

23. Name the vertex of $\angle 7$.
24. Write another name for $\angle 4$.
25. Name the sides of $\angle 2$.
26. Name a pair of opposite rays.
27. SIGNS A sign at West High School has the shape shown. Measure each of the angles and classify them as right, acute, or obtuse.


## Example 5

Refer to the figure below. Name all angles that have $Q$ as a vertex.

$\angle O Q N, \angle N Q P, \angle P Q R, \angle R Q S, \angle S Q O, \angle O Q P, \angle N Q R$, $\angle P Q S, \angle O Q R$

## Example 6

In the figure above, list all other names for $\angle 1$.
$\angle N O Q, \angle Q O N, \angle M O Q, \angle Q O M, \angle M O R, \angle R O M, \angle N O R, \angle R O N$

## 1-5 Angle Relationships

For Exercises 28-30, refer to the figure below.

28. Name an angle supplementary to $\angle T V Y$.
29. Name a pair of vertical angles with vertex $W$.
30. If $m \angle S X W=5 x-16$, find the value of $x$ so that $\overline{S X} \perp \overline{W Y}$.
31. PARKING The parking arm shown below rests in a horizontal position and opens to a vertical position. After the arm has moved $24^{\circ}$, how many more degrees does it have to move so that it is vertical?


## Example 7

Name a pair of supplementary angles and a pair of complementary angles in the figure below.


Sample answers:
Supplementary angles: $\angle R X A$ and $\angle R X D$
Complementary angles: $\angle R X C$ and $\angle C X D$

## Two-Dimensional Figures

Name each polygon by its number of sides. Then classify it as convex or concave and regular or irregular.
32.

33.

34. Find the perimeter of quadrilateral $A B C D$ with vertices $A(-3,5), B(0,5), C(2,0)$, and $D(-5,0)$.
35. PARKS Westside Park received 440 feet of chain-link fencing as a donation to build an enclosed play area for dogs. The park administrators need to decide what shape the area should have. They have three options: (1) a rectangle with length of 100 feet and width of 120 feet, (2) a square with sides of length 110 feet, or (3) a circle with radius of approximately 70 feet. Find the areas of all three enclosures and determine which would provide the largest area for the dogs.

## Example 8

Name the polygon by its number of sides. Then classify it as convex or concave and regular or irregular.


There are 6 sides, so this is a hexagon. If two of the sides are extended to make lines, they will pass through the interior of the hexagon, so it is concave. Since it is concave, it cannot be regular.

## Example 9

Find the perimeter of the polygon in the figure above.

$$
\begin{aligned}
P & =s_{1}+s_{2}+s_{3}+s_{4}+s_{5}+s_{6} & & \text { Definition of perimeter } \\
& =7+7+9+6+6+4 & & \text { Substitution } \\
& =39 & & \text { Simplify. }
\end{aligned}
$$

The perimeter of the polygon is 39 units.

